Mathematical education
Математическое образование

UDC 372.851

THE ROLE OF DIFFERENTIAL EQUATIONS IN PHYSICAL EXERCISE

N.B. Otojonova[1], D.B. Otojonova[2]
[1] Senior Lecturer Department of Mathematics, [2] Candidate of Philology (PhD)
Tashkent Regional Chirchik State Pedagogical Institute, Uzbekistan

Abstract. *The article discusses the role of differential equations in solving physical exercise. It is shown that with the help of differential equations many physical exercises are easily solved.*
Keywords: *mathematics, differential equations, physical exercise, physical waves.*

Today in our country a lot of attention is paid to the science of mathematics. In this regard, it is possible to cite the decisions taken at the national level. For example, the President of the Republic of Uzbekistan dated July 9, 2019 "On measures to support the further development of mathematics education and science, as well as measures to radically improve the activities of the Institute of Mathematics named after V.I. Romanovsky of the Academy of Sciences of Uzbekistan" Resolution No. 4387 and Resolution No. 4708 of May 7, 2020 "On measures to improve the quality of education and research in the field of mathematics" provide for the implementation of a number of tasks [3, 4]. The implementation of such tasks today is one of the most pressing problems [2]. We all know that a number of areas of mathematics are used in the implementation of all tasks in human life.

Mathematical analysis has evolved in close connection with the natural sciences, especially physics and mechanics, since its inception as the analysis of variables. The need to develop the science of physics, the need for quantitative study of motion and variable processes has led to the emergence and formation of basic concepts of differential and integral calculus [1].

In the eighteenth century, the theory of differential equations emerged as a science independent of mathematical analysis. His achievements are connected with the names of the Swiss scientist I. Bernoulli, the French mathematician J. Lagrange, and especially L. Euler. The initial period of development of the theory of differential equations should be found through the starters. However, it has been shown that there are very few equations that integrate quickly. Even very simple first-order equations may not be integrated into squares.

To explain the differential equation, one of the basic concepts is to study a physical process. To create a physical hypothesis based on this experience, to write a physical hypothesis in a mathematical form, to obtain a mathematical solution to this problem, and finally to interpret the conclusions from a physical point of view. Such an approach to the study of natural phenomena was first proposed by the Italian scientist Galileo (1564-1642). It was skillfully used by Newton, one of the founders of mathematical analysis. The mathematical explanation of the laws of physics was possible only with the advent of mathematical analysis, and the language of mathematical analysis allowed it. In almost all cases, the laws of physics describes the relationship between the rates of change. These laws are represented by equations involving unknown functions and their derivatives. Such equations are called differential equations. They have emerged as mathematical expressions of a number of laws of physics. The processes described by such laws lead to the study of the properties of solutions of differential equations. We see this in the following examples.

An object whose temperature rises to y_0 degrees (for example, a metal plate) is placed in a very large vessel filled with zero-degree air at time $t = 0$. It is known that an object begins to cool and its temperature is a function of time t. We denote this function by $y(t)$.

According to Newton's law of cooling, the rate of change of body temperature $\frac{dy}{dt}$ is proportional to the difference between the temperature of the product and the surrounding environment, in our case $y(t)$. Accordingly, the relation $\frac{dy}{dt} = ky$ at each instant of time is appropriate (the coefficient of the k-body depending on the material receives a "minus" sign as the temperature decreases). $\frac{dy}{dt} = ky$ in the form of a differential equation. The relationship is a mathematical expression of the law of cooling. It represents the relationship between a function of time and its product $\frac{dy}{dt} = ky$. The ratio is also called the mathematical model of the process under consideration. It is known that the solution of the differential equation is to find all the functions $y(t)$.that make the equation real. All solutions of the above differential equation are found by the formula $y = Ce^{-kt}$ (C is an arbitrary constant). Find the solution of the differential equation is always associated with the operation of integration. Therefore, instead of the word "solution" is often used the term "integration" of the differential equation. The phenomenon we are studying, that is, the cooling process of the body only at time $t = 0$ y_0 is only interested in a solution that accepts the value. Substituting $t = 0$ into the above formula, we find that $C = y_0$. Hence, the law of cooling can be given a final expression in the form $y(t) = y_0 e^{-kt}$. It can be seen that the temperature of a body decreases with time according to the exponential law and tends to the ambient temperature. $y(0) = y_0$ condition is taken as the initial condition, which allows to separate one from an infinite set of solutions.

The differential equation seen above represents the fact that the rate of change of the function $\frac{dy}{dt} = ky$ is proportional to the function itself (with k-coefficient). A similar relationship is observed in other phenomena of nature. For example, the dependence of atmospheric pressure on sea level The decrease is proportional to the magnitude of the pressure. Another example is radioactive decay: the rate of decrease in the mass of a radioactive substance is proportional to the amount of that substance. Hence, atmospheric pressure y satisfies the equation $\frac{dy}{dt} = ky$ as a function of altitude t above sea level and the mass of radioactive matter as a function of time t. From this, we see that exactly one differential equation can be used as a model of absolutely different mathematical phenomena.

The differential equation seen above represents the fact that the rate of change of the function $\frac{dy}{dt} = ky$ is proportional to the function itself (k - coefficient). A similar relationship is observed in other phenomena of nature. The decrease is proportional to the magnitude of the pressure. Another example is radioactive decay: the rate of decrease in the mass of a radioactive substance is proportional to the amount of that substance. Hence, atmospheric pressure y satisfies the equation $\frac{dy}{dt} = ky$ as a function of altitude *t* above sea level and the mass of radioactive matter as a function of time *t*. From this we see that exactly one differential equation can be used as a model of absolutely different mathematical phenomena.

In the next example, let's look at a small sphere of mass *m*. Let's attach a spring to it horizontally. The other end of the spring is attached to it. If the ball is moved slightly along the axis, a tensile force *F* tends to return it to equilibrium. According to Hook's law, this force is proportional to the magnitude of the displacement *x*, $F = -kx$ (k is the elastic property of the spring. characteristic positive constant, the sign "minus" indicates that the effect of force is inversely proportional to the direction of elongation). According to Newton's second law, the force acting on a body of mass m is equal to the product of mass and acceleration: *F = ma*.

If *x (t)* is the momentum of the sphere, its acceleration is expressed by the second product $x''(t)$. Thus, the motion of a balloon under the influence of elastic forces can be expressed by the differential equation $mn'' = -kx(t)$. It's more

$$x''(t) + \omega^2(t) = 0, \tag{1}$$

where ($\omega^2 = \frac{k}{m}$) is written in the form. This equation is called the differential equation of harmonic oscillations. Its optional solution can be written as follows:

$$x(t) = Acos(\omega t + \varphi), \tag{2}$$

where A and φ are arbitrary variables. Quantities characterized by such an equation are called harmonic oscillations. Their period consists of a periodic magnitude equal to $t = \frac{2\pi}{\omega}$, the coefficient A is called the oscillation amplitude. (1) does not fully define the motion of the expression. This motion can vary depending on the displacement of the balloon at time $t = 0$ to x_0 and how fast $\vartheta = x'(0)$ is pushed, i.e. the initial values. For example, if the initial velocity is 0, the motion of the balloon

$$x(t) = x_0 cos\omega t \tag{3}$$

subject to the law. The differential equation given above is only a mathematical expression of the law of motion under the action of an elastic force. If we consider the motion of a balloon in a resistive medium and assume that in addition to the force of elasticity on the balloon is also a force of resistance proportional to velocity, then the differential equation of such motion is

$$mx''(t) + cx'(t) + kx(t) = 0 \tag{4}$$

The solutions of these equations are no longer periodic functions, but of oscillations of variable amplitude. They are called extinct oscillations. If an unknown function depends on only one variable, the differential. The equation is called simple, and the above equations are among such equations. The order of the differential equation is the highest order of the products involved in it. For example, $\frac{dy}{dt} = -ky$ means that the equation is first-order, $x''(t) + \omega^2 x(t) = 0$ is second-order.

If an unknown function depends on more than one variable, the special product of the differential equation is called the differential equation. Such equations include the vibration of a membrane (a thin elastic plate), the distribution of heat in an environment, the motion of a satellite, and so on. It follows that differential equations are the basic mathematical apparatus of the natural sciences, especially physical phenomena. Similar equations are used in physics, astronomy, aerodynamics, chemistry, economics, biology, and medicine.

The differential equation of the first order $\frac{dx}{dt} = f(x, t)$ is interpreted in simple geometry. If $x = \varphi(t)$ is the solution of the first order equation, then our equation $x = \varphi(t)$ gives the value of the product at each point of the line, that is, the value of the tangent of the angle of inclination. Thus, at each point in the domain of the function, the angular coefficient of the solution is given, and in this case, the area of the directions is given. From a geometric point of view, the area of directions is usually expressed in unit vectors. The area of directions of the differential equation $\frac{dx}{dt} = t^2 + x^2$ is shown. The solution of the differential equation is a line that moves in the direction of the field at each of its points. It is called an integral line. It allows you to clearly see what the integral lines of the equation are.

Now, let's look at the laws of free fall of bodies. Let's lift a small rock and then drop it from a state of rest. Let t be the time elapsed from the beginning of the day, and $s(t) - t$ be the time elapsed from time to time. Galileo experimentally found that the s (t) bond had the following simple form:

$$s(t) = \frac{1}{2} g t^2, \tag{5}$$

where t is the time in seconds, and g is a physical constant of about 9,8 m/s^2

It is natural for the motion of a free-falling object to be uneven. The rate of descent ϑ gradually increases. But what does the $\vartheta(t)$ relation look like? Obviously, knowing the relation s (t), that is, the law of motion of a moving body, we must be able to express the velocity $\vartheta(t)$ as a function of time. Let's try to find how ϑ depends on t. Consider: $\vartheta(t)$ is the moment t we want to know the value of the velocity. Let t be a small time interval h. It is known that an object falling in time h travels a path equal to $s(t+h)-s(t)$ If h is very small, the velocity of the body cannot change significantly over time h, so at a small value of h

$$s(t+h)-s(t)\approx\vartheta(t)\cdot h \tag{6}$$

or

$$\frac{s(t+h)-s(t)}{h}\approx\vartheta(t) \tag{7}$$

can be considered. Then the smaller the approximate equation h (the closer h is close to zero), the more accurate it is. Hence, the magnitude of the velocity $\vartheta(t)$ at t is given by o in the time interval from t to $t+h$. The approximate equation (7) representing the average velocity can be thought of as the limit of the ratio to the left of when h tends to zero. This is in mathematical language

$$\vartheta(t)=\lim_{h\to0}\frac{s(t+h)-s(t)}{h} \tag{8}$$

Let us now perform the calculations given in relation (8) from the relation (5) found by Galileo. First of all, let's do some simple calculations:

$$s(t+h)-s(t)=\frac{1}{2}g(t+h)^2-\frac{1}{2}gt^2=$$

$$=\frac{1}{2}g(t^2+2th+h^2)-\frac{1}{2}gt^2=gth+\frac{1}{2}gt^2. \tag{9}$$

Now divide this expression by h,

$$\frac{s(t+h)-s(t)}{h}=qt+\frac{1}{2}qh \tag{10}$$

we create an equation. When h tends to zero, the second addition to the sum written on the right also tends to zero, while the first remains unchanged, more precisely, h does not depend on the magnitude, so in this case

$$\vartheta(t)=\lim_{h\to0}\frac{\frac{1}{2}g(t+h)^2-\frac{1}{2}gt^2}{h}=gt. \tag{11}$$

Thus we find that the law of variation of the velocity of a freely falling body is $(t)=gt$. $\vartheta(t)=\lim_{h\to0}\frac{s(t+h)-s(t)}{h}$ note that the formula gives both the definition and the calculation rule of $\vartheta(t)$ the instantaneous rate of change of the function s (t) at the same time. Since the velocity $\vartheta(t)$ is itself a function of time, it was possible to ask the question of its rate of change. In physics, the rate of change of speed is called acceleration. Thus, if $\vartheta(t)$ is a function of time velocity, then $\vartheta(t)=\lim_{h\to0}\frac{s(t+h)-s(t)}{h}$ as in the derivation of the formula. This is for the instantaneous acceleration a (t) at time t

$$a(t) = \lim_{h\to 0} \frac{\vartheta(t+h)-\vartheta(t)}{h} \quad (12)$$

we get the expression.

The identification of the above facts was a major impetus for the development of the theory of differential equations. This theory deals with the development of methods that allow to determine the properties and nature of the solutions of differential equations. It can be seen from the examples that the branch of differential theory of mathematical analysis is the mathematical representation of many natural sciences, including physical problems. In addition, some problems in the natural sciences, such as chemistry and biology, can be solved only with the help of differential equations. It follows that differential equations are an important mathematical apparatus for solving problems in the natural sciences.

REFERENCES

1. Azamov, A. Young mathematician / A. Azamov // Encyclopedic dictionary. – Tashkent. – 1991. – 478 p.
2. Begaliev, J.U. The role of physics in career guidance / J.U. Begaliev, I.U. Tadjibaev, Sh.N. Usmonov // Pedagogy & Psychology. Theory and practice. – 2020. – No. 4 (30) (in press) (in Russian)
3. Resolution of the President of the Republic of Uzbekistan dated May 7, 2020 No 4708 "On measures to improve the quality of education and development of research in the field of mathematics" // lex.uz
4. To the President of the Republic of Uzbekistan "About measures of the state support of further development of education and sciences of mathematics, and also radical improvement of activities of Institute of Mathematics of V.I. Romanovsky of Academy of Sciences of the Republic of Uzbekistan" Resolution No. 4387 of July 9, 2019 // lex.uz

Материал поступил в редакцию 05.08.20

РОЛЬ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ В ФИЗИЧЕСКИХ ЗАДАЧАХ

Н.Б. Отожонова[1], Д.Б. Отожонова[2]
[1] старший преподаватель кафедры математики, [2] кандидат филологических наук (PhD)
Чирчикский государственный педагогический институт Ташкентской области, Узбекистан

Аннотация. *В статье рассматривается роль дифференциальных уравнений в решении физических задач. Показано, что с помощью дифференциальных уравнений легко решаются многие физические задачи.*

Ключевые слова: *математика, дифференциальные уравнения, физические задачи, физические волны.*