STRESS REDUCTION METHODS FOR CENTRIFUGAL IMPELLERS

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ABSTRACT: Centrifugal compressors are a crucial component of jet engines and the current trends of increasing propulsion systems performance include increasing the pressure ratio so that the mechanical demands to which the rotor is subjected also increase. The aim of the paper is to highlight how geometry optimization can lead to weight reduction while obtaining a very good mechanical behavior (under mechanical stress and vibration).

KEYWORDS: centrifugal compressor, jet engine, weight optimization, mechanical stress, vibration

NOMENCLATURE

\begin{tabular}{ll}
  f & frequency \\
  f_e & excitation frequency \\
  f_s & static frequency \\
  g & blade root maximum thickness \\
  n_s & rotational speed in rot/s \\
  sc & safety coefficient \\
  A & area \\
  D & cylindrical rigidity \\
  E & modulus of elasticity \\
  FEA & finite element analysis \\
  FEM & finite element method \\
  J & moment of inertia \\
  K & number of excitation factors \\
  K_s & coefficient for static frequency \\
  N_{s,t} & total number of sections \\
  R_1 & boss main radius \\
  UAV & unmanned aerial vehicle \\
  \alpha & thermal expansion coefficient \\
  \beta & coefficient for vibration modes \\
  \rho & density \\
  \sigma_0 & yield stress \\
  \sigma_{1,2,3} & principal stress in direction 1,2 or 3 \\
  \sigma_{eq} & equivalent stress \\
  \sigma_{max} & maximum stress \\
  \sigma_r & radial stress \\
  \sigma_t & tangential stress \\
  \omega & angular velocity \\
  [ i ] & index corresponding to section i \\
\end{tabular}

1. INTRODUCTION

In the field of modern aviation, the main applications of centrifugal compressors are microjet engines, APUs or turboshaft engines (which require relatively low flow rates compared to the high dilution turbofan engines). Numerous studies are being conducted on impeller optimization methods, as this is the element with the greatest influence in the centrifugal compressor assembly [1]. Regarding compressors with low flow rates, around 1 kg/s [2], the tendency is to increase the pressure ratio (reaching values of 10) which implies the use of very high rotational speeds leading to transonic peripheral speeds at the impeller tip [3]. Since in the field of aviation safety is a priority, each component of the engine must be well designed and tested to ensure its functionality at the desired parameters. For compressors with usual pressure ratio and flow rates, most development directions are aimed at increasing efficiency, reducing component weight, increasing operating range, lowering production costs, while maintaining acceptable safety limits [4], [5]. This paper aims to lead to some results (at least qualitative) that can be taken into account when designing a centrifugal rotor. The optimization from the design stage is very important since testing the centrifugal compressors on the test bench is very expensive,

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taking into account the fact that a modal analysis (as opposed to the aerodynamic resistance and performance tests) requires a lot more advanced equipment (specific to optical interferometry [6]). Thus, the most convenient analysis that can be performed is a numerical one, using a FEM analysis, qualitatively confirmed by a theoretical analysis combined with semi-empirical formulae.

2. MAIN FORCES ACTING ON THE IMPELLER

The angular velocities combined with the torsion of the blades lead to the occurrence of both tensile and bending forces. Using radial blades is the simplest way to reduce the overall load on the impeller because the forces acting on them are generated by the pressure difference (pressure side – suction side) and by the main component of the centrifugal force acting in the radial direction (low bending force). In terms of aerodynamic performance, the radial blades are slightly disadvantageous, but for a simplified structural calculation they are desirable. In the current paper, a geometry with radial blades (except for the anterior part) will be used in order to better correlate the results obtained from the finite element analysis with the theoretical calculation. If the impeller is considered to be a rotating disk, the forces acting on an area element are represented in fig. 1 (b).

Radial and tangential stresses can be written in differential form [7]

\[
\begin{align*}
\sigma_R = \frac{dR}{R} - \sigma_R \left( \frac{dR}{R} + \frac{db}{b} \right) - \rho \omega^2 R^2 \frac{dR}{R} \\
\sigma_T = \sigma_T \left( -\frac{dR}{R} + \frac{dE}{E} \right) + \sigma_T \left( -\mu \frac{db}{b} - \mu \frac{dE}{E} + \frac{dR}{R} \right) - \mu \rho \omega^2 R^2 \frac{dR}{R} - Ed(\alpha t)
\end{align*}
\]

and the equivalent stress can be found using

\[
\sigma_{ech} = \sqrt{\sigma_R^2 + \sigma_T^2}
\]

The analytical calculation of vibration implies the consideration of the radial blade with a cantilever plate and the semi-empirical formula can be used [7]

\[
f_{st} = \frac{17.5}{l^2} \left( \frac{E \cdot J}{\rho \cdot A} \right)^{\frac{1}{2}}
\]

and for the rotating disk, the frequencies corresponding to a certain number of diameters and circles of nodes are determined by summing on the sections considered in the structural calculation using [7]
\[
\frac{1}{f^2} = K_f \sum_{i=1}^{N_{tr}} \frac{1}{f_i^2}
\]

\[
f_i = \frac{\beta}{2\pi \cdot R_{mi}} \sqrt{\frac{D_i}{\rho \cdot b_i}}
\]

Unlike analytical calculation, finite element analysis no longer uses semi-empirical or empirical coefficients but is based on solving a system of differential equations (static equations - Navier-Cauchy equations, geometric equations - Saint Venant equations, physical equations - generalized Hooke’s Law) which eliminates the dependency of the conditions used to determine those coefficients.

The von Mises equation (for principal stresses) used in FEA is presented in relation (5). [8],[9]

\[
\sigma_v = \sqrt{\frac{1}{2} \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]}
\]

3. VIBRATION AND STRESS EVALUATION

For the vibration and stress evaluation, the centrifugal rotor described in [1] is used, which is part of a compressor that is supposed to equip a microjet engine for a UAV. The impeller has a diameter of 250mm and is designed to operate at 40000 rpm. The FEM analysis was performed in several steps. For the proper use of computer resources, only a rotor sector was used which consists of a blade and a splitter. Because it is an aviation specific centrifugal compressor, the aerograde materials used for the finite element analysis are the following: 7075 aluminum alloy and Ti6Al4V titanium alloy whose properties are presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>7075 Aluminum Alloy</th>
<th>Unit</th>
<th>Ti6AI4V Titanium Alloy</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>2810</td>
<td>kg/m³</td>
<td>4430</td>
<td>kg/m³</td>
<td></td>
</tr>
<tr>
<td>( E )</td>
<td>71.7</td>
<td>GPa</td>
<td>114</td>
<td>GPa</td>
<td></td>
</tr>
<tr>
<td>( \sigma_{02} ) (at 20°C)</td>
<td>572</td>
<td>MPa</td>
<td>950</td>
<td>MPa</td>
<td></td>
</tr>
<tr>
<td>( \sigma_{02} ) (at 150°C)</td>
<td>380</td>
<td>MPa</td>
<td>700</td>
<td>MPa</td>
<td></td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.33</td>
<td>-</td>
<td>0.31</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

As is well known in the field of numerical simulations, the use of a periodic element (disk sector) does not introduce additional errors to the final result [11]. For the analyzed sector (with the symmetry condition applied at the cut faces), the two cylindrical channels of diameter 5mm and depth 25mm were not taken into account. For the vibrational analysis, the complete geometry (once the final solution - material and geometry was established), to which the notches were added.

The boundary conditions are represented by the cylindrical symmetry on the radially cut faces, the axial fixation on the nut fitting surface and the cylindrical fixation on the shaft surface and the speed at which the rotor was calculated is 40000 rpm.

In the work [1] it was tried (in the first place) the simulation of a sector consisting of a blade and a splitter (without hub) but the fixation at the base introduced additional stress that does not appear normally. FEA were performed using the specialized programs PATRAN (for pre and post processing) and NASTRAN (for the calculation itself).
4. OPTIMIZATION CRITERIA

Although the stress that appears in the rotor is relatively small, it is observed that there is a region where the allowable stress is exceeded (according to table 1 and fig. 2). Because the centrifugal force is directly proportional to the density, it is obvious that changing the material does not greatly alter the stress distribution. It is correct that the use of other material with better mechanical properties would lead to a better $\frac{\sigma_A}{\sigma_{\text{max}}}$ ratio but the use of the material is not fully exploited. In order to do so, two modifications can be made to the impeller: it can either be cut from the base of the rotor (which leads to a weight reduction) -fig. 3a or a boss (protrusion) (fig. 3b and 3c) can be made in the area of high stress.

Applying cutting solution (a), the stress that appears in the bladed sector is shown in fig. 4.

For the rotor made of aluminum alloy, a different distribution of material stress is observed compared to the initial geometry, but the maximum stress that appears is obviously above the creep limit. Changing the material for the same new geometry results in a $\frac{\sigma_A}{\sigma_{\text{max}}}$ ratio over 1 but this is not satisfactory.
Stress reduction methods for centrifugal impellers

On the one hand the alternative with the boss (fig. 5) is better than the one with the cut, in the case of aluminum alloy, on the other hand for the titanium alloy the results are very good but at the expense of increasing the weight of the rotor and the overall price.

Summarizing the obtained results, it is observed in table 3 that there are geometries that satisfy the resistance condition (safety coefficient > 1). In addition to the numerical results, the maximum stress resulting from the analytical calculation (table 2) was added by using the method of solving the differential equations (1) which is described in [7].

Table 2. Maximum stress in baseline rotor (finite element vs theoretical)

<table>
<thead>
<tr>
<th>Geometry</th>
<th>σA [MPa]</th>
<th>analytical</th>
<th>finite element</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>σmax [MPa]</td>
<td>sc [-]</td>
</tr>
<tr>
<td>Baseline Al alloy rot</td>
<td>380</td>
<td>247.5</td>
<td>1.53</td>
</tr>
<tr>
<td>Baseline Ti alloy rot</td>
<td>700</td>
<td>404.6</td>
<td>1.73</td>
</tr>
</tbody>
</table>

Table 3. Maximum stress in modified impeller

<table>
<thead>
<tr>
<th>Geometry</th>
<th>σA [MPa]</th>
<th>σmax [MPa]</th>
<th>sc [-]</th>
<th>Rotor weight [kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Al alloy rot</td>
<td>380</td>
<td>424</td>
<td>0.89</td>
<td>2.017</td>
</tr>
<tr>
<td>Baseline Ti alloy rot</td>
<td>700</td>
<td>668</td>
<td>1.04</td>
<td>3.18</td>
</tr>
<tr>
<td>Al alloy cut impeller</td>
<td>380</td>
<td>429</td>
<td>0.88</td>
<td>1.878</td>
</tr>
<tr>
<td>Ti alloy cut rot</td>
<td>700</td>
<td>678</td>
<td>1.03</td>
<td>2.961</td>
</tr>
<tr>
<td>Al alloy boss rot (R=5mm)</td>
<td>380</td>
<td>356</td>
<td>1.06</td>
<td>2.45</td>
</tr>
<tr>
<td>Al alloy boss rot (R=30mm)</td>
<td>380</td>
<td>420</td>
<td>0.9</td>
<td>2.45</td>
</tr>
<tr>
<td>Ti alloy boss rot (R=30mm)</td>
<td>700</td>
<td>484</td>
<td>1.44</td>
<td>3.862</td>
</tr>
</tbody>
</table>

To verify that the most advantageous solution is fully usable, the radial and axial displacements for the model made of titanium alloy are shown in figure 6. The maximum radial displacement is observed to be about one tenth of a millimeter, which is a value in the usual range. The minimum axial displacement is (as expected) on the fixing surface with the nut and maximum in the area of minimum thickness (at the top of the blade).
It is necessary to determine the static and dynamic frequencies because at certain rotor speeds it can happen that an excitation frequency equals the vibrational frequency of the rotor and the resonance phenomenon occurs. For the optimum solution of the rotor (the Ti rotor with the boss), the static and dynamic frequencies of the rotor were determined analytically (using relations (3) and (4)) and numerically (using FEA) and represented in Table 4. For the analytical determination of static frequencies, the model of the disc with a central hole, fixed at the center and free at the edges [7] was used, which corresponds to the boundary conditions imposed in the FEM analysis. Figure 7 shows the modes of vibration using the Lanczos method (with normalization parameter set for unit amplitude). The method uses eigenvalues so that the obtained values do not offer a quantitative value but a qualitative representation, because each eigenvector multiplied by a constant gives an eigenvector as well [10].

![Vibration modes](image)

**Fig.7. Impeller vibration modes (a) 1757.3Hz, (b) 3312.2Hz, (c) 3787.6Hz, (d) 3891.5Hz, (e) 3979.9Hz, (f) 4115.7Hz [1]**

<table>
<thead>
<tr>
<th>Nodal circles</th>
<th>Nodal diameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>2997.2 Hz</td>
</tr>
<tr>
<td>1</td>
<td>16745.1 Hz</td>
</tr>
</tbody>
</table>

**Table 4. Static frequencies analytically determined**

<table>
<thead>
<tr>
<th>Percentage of max rpm</th>
<th>Mode 1 (blades)</th>
<th>Mode 2.1 (disk torsion)</th>
<th>Mode 3 (first nodal diameter)</th>
<th>Mode 4 (second nodal diameter)</th>
<th>Mode 2.2 (zero nodal diameters)</th>
<th>Mode 5 (third nodal diameter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1757.3</td>
<td>3312.2</td>
<td>3787.6</td>
<td>3891.5</td>
<td>3979.9</td>
<td>4115.7</td>
</tr>
<tr>
<td>50</td>
<td>1920.2</td>
<td>3347</td>
<td>3840.7</td>
<td>3960.9</td>
<td>4029.2</td>
<td>4205.9</td>
</tr>
<tr>
<td>100</td>
<td>2069.2</td>
<td>3381.5</td>
<td>3892.5</td>
<td>4028.6</td>
<td>4077.4</td>
<td>4293.7</td>
</tr>
<tr>
<td>110</td>
<td>2097.6</td>
<td>3388.3</td>
<td>3902.7</td>
<td>4041.9</td>
<td>4086.9</td>
<td>4311</td>
</tr>
</tbody>
</table>

**Table 5. Static and dynamic frequencies**

In order to justify the need for vibration analysis, it is necessary to draw the frequency-rotational speed diagram (Campbell diagram) [12], [13]. By determining the excitation frequencies using the relation (6), where \( K \) represents the number of excitatory factors (number of mounting/stator blade or number of injectors from the combustion chamber), critical speeds can be determined and precautionary measures can be taken to avoid an operating regime with high vibration amplitudes, possibly long-term destructive. [7]

\[
 f_e = K \cdot n_e 
\]  

(6)
Because the number of excitatory factors may vary, the usual values for microjet engines were chosen: 3 and 4 for stator blades and 5-7 for the number of injectors and the excitation frequencies determined using these values are represented in the Campbell diagram (Figure 8).

![Campbell diagram](image)

From the analysis of the diagram represented in figure 8 it can be concluded that, for the used compressor geometry, some of the critical speeds are found in the continuous operating range. For \( K = 6 \) there is a resonance in the idle domain and another one near the maximum regime. For \( K = 4 \) there is a resonance at the corresponding speed of 75\% of the maximum speed and \( K = 3 \) and \( K = 5 \) are dangerously close to the maximum speed zone. It should be taken into account that when designing the whole engine assembly, using a number of elements corresponding to those previously mentioned must be avoided.

5. CONCLUSIONS

An optimization can be made regarding the centrifugal rotor by cutting the bottom of the impeller which leads to a 7\% weight reduction compared to the full rotor and a more uniform distribution of high stress intensity but the maximum stress value remains close to the same value as that of the baseline. This procedure cannot be applied to all materials, observing the analyzed case that it is also necessary a material with special properties which leads to higher costs. The variant with boss is less beneficial in terms of weight but allows the use of materials with weaker mechanical properties. In the most favorable case (Ti alloy rotor), a maximum stress decrease of 27.5\% was obtained for a weight gain of 21\%. It was also observed that the simplified analytical calculation leads to an optimistic result (with high safety coefficients) but for the real case, with complicated geometry, leads to the appearance of areas that cannot be highlighted by the applied calculation procedure. It is obvious that the best solution for the current application was the one with the boss followed by the cutting solution and the baseline. A combination of the first two solutions will be analyzed in future work.

Regarding vibration analysis, it was observed that a very small thickness of the blade root leads to the appearance of some vibration forms on the blade before it appears on the disk. This should be avoided because the vibration of the blades at low frequencies, combined with their high flexibility can lead to quite large amplitudes that can favor material fatigue and cracks in the area of high material stress.

Future work will be focused on a parametric study on the influence of different material removal or addition at the bottom of the disk for different common materials used in centrifugal compressors along with a material fatigue analysis.

ACKNOWLEDGEMENT

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REFERENCES


[5] Sorokes J. M., “Selecting a Centrifugal Compressor”, June 2013, American Institute of Chemical Engineers (AIChE) and Dresser-Rand


