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COMPARING REDUCED FORM VS. STRUCTURAL MODELS IN MEASURING AND MANAGING CREDIT PORTFOLIO RISKS - A COMPREHENSIVE META – ANALYSIS

Introduction

The precise measurement of credit risk is crucial for financial institutions. As they are not only obliged to fulfil the legal capital and processual/management requirements from regulators like EBA and ECB (or the respective NCAs) captured through the CRR II and CRD V regulations in the EU but also have an inherent business interest (including these of stakeholders and shareholders) regarding managing their loan portfolios and corresponding risks (of default and credit migration) the use of advanced credit risk models is an industry-wide standard.

In other jurisdictions like the U.S. or Switzerland, the situation is - due to the international Basel III finalization accords and respective derived national acts – broadly comparable. Nevertheless, there are aspects and underlying factors in credit risk modelling that are still quite unilluminated – especially a comprehensive meta-analysis researching the existing papers comparing the models. The application of a superior model gives a bank a competitive advantage in managing its credit related risks and is eventually lowering the capital needed to cover potential losses (better risk bearing capacity). Hence, an in-depth comparison of credit portfolio risk models has substantial impact and is the main goal of this publication. The null hypothesis – which we disprove later - is that the industrial models CreditMetrics® and CreditRisk+® perform equally well – especially among corporate creditors.

The author devotes himself to the task of a precise comparison and further briefly explains the strengths and shortcomings of the models as well as the application to certain portfolio or customer segments before.

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There are currently three main models to measure Credit Portfolio Risk:

1. Structural, Merton-type migration models which are asset value based.
2. Hazard models also known as intensity rate models or reduced form models stemming from actuarial backgrounds.
3. Macroeconomic models or econometric, macro-factor models - which are rather similar to structural ones.

The structural models were first presented by Merton (1974) and further developed by Leland and Toft (1994, 1996), by Anderson and Sundaresan (1996) and later by Jarrow (2011).

Since the 1990s, actuaries and statisticians at Credit Suisse First Boston (CSFB) have developed new methods of risk management, at the end of which they presented CreditRisk+®.

Both mainly used types of Credit Portfolio models, structural and hazard rate (also known as reduced - form) models are categorized here. The third type is the macro-factor based regressive one with a logit (e.g., Wilson-Model or probit - type) link function and mainly used in loan-and-savings institutes (like German "Sparkassen" with the CPV – Credit Portfolio View ® application, originally developed in 1998 by McKinsey and Company). It links defaults to macroeconomic factors such as the growth rate of the GDP, the long-term interest rate, the unemployment rate, the exchange rates or the public spending. As it can be transformed to and sub-summarized under model type 1. it is not further investigated in the meta-research of this paper.

I begin with a qualitative and descriptive part as first goal, which is followed by the segment comparing the results of the existing literature.

The first and most obvious differences are inherent in the models themselves and are encoded in the way a default (event) is described and on the assumptions the models have as well as their underlying mathematical way of modelling credit risk itself.

The following table is featuring these differences.

Table 1. **Qualitative comparison of intensity rate and structural models**

Model	**Intensity rate model**	**Structural model**
Synonyms used, equivalent models	Hazard rate model, reduced form, actuarial model, (mixed) Poisson-Gamma-distribution model, credit default model	Merton model, enterprise value model, asset-value based model, credit migration model

Economic background, accessibility	No direct economic derivation, hard to access and imagine, economically atheoretic but flexible	Direct economic derivation and dependence on one (in case of one-factor model, "business cycle"/GDP growth) or more economically feasible factors, good to access
Default	Defaults appear randomly, exogenously defined, no hypothesis on the causes of default of a company. This is directly linked to the information set known to the observer.	Default (and its causes) are (totally) pre-defined by a company's debt and its asset structure and movement
Information (requirement)	No further (i.e. more than market information) information is required, very realistic	(Full) information (about the company's capital) is required and transparent, idealized transparent market
Origins	Valuation of companies' assets/capital-structure, Option theory	Insurance mathematics, actuarial and stochastic
Generality	Not too general, danger of "overfitting"	Very high and very general
Calibration	Very exact calibration is possible	Very exact calibration is possible
Extensions	Migration mode models, stochastic interest rates, stochastic correlations (in the basic version - the number of defaults over a period is independent from that of any other period) – e.g. by Duffie and Singleton, stochastic recoveries, linkage through Copulas (though CID most popular)	Multi-factor models (by LI et al.), randomness in defaults, different OIS-risk free curves and multi-curve discounting (post-crisis pricing cf. "Big Bang in finance"), stochastic correlation and migration matrices (in the basic version - the number of default over a period is independent from that of any other period), stochastic recoveries (yet not much more precise) and stochastic interest rate (Longstaff and Schwartz, 1995), optimal permanent capital (Leland and Toft, 1996), variable time of the threshold of default (Collin-Dufresne and Goldstein,

		2001), unfinished accounting information (Duffie and Lando, 2001) and the risk of the events of defect (Driessen, 2005). It was also extended for linkage through Copulas (though not common)
Company associated with it	Credit Suisse First Boston	JP Morgan

Source: own illustration.

Having laid out these differences, one can directly summarize the following results:

- The structural models are more approachable and economically feasible compared to reduced form models – e.g. described detailed by Derbali[1] - whereas
- Reduced form models are easier to implement, they need less data, storage and computer performance.

One has to further notice as we want a reliable comparison, in most quantitative studies one just uses the structural/Merton model in default mode as to compare it to the hazard rate type (which is a default mode model). Hence, just the jump to default is considered whereas most structural models (e.g. the CreditMetrics® implementation) are used as migration (mode) models, calculating all possible rating grade migrations (i.e. from AAA to AA- or from BB+ to C) not just the one(s) to default (D) – we therefore substantially limit the setting of the model. While it is possible to introduce migration stages in hazard rate models as well (and it is not too technical), this often leads to unstable results and is not common in practice.

Reviewing the literature one yields the results stated in the following:

The first important comparison of credit portfolio models was done by Koyluoglu and Hickman (1998)[2]. The primary aim of this comparison was to show that the underlying ideas, theories and results of the three Credit Portfolio models CreditMetrics®, CreditRisk+® and Credit Portfolio View® are similar - and by putting them within a single general framework and harmonizing their parameters Koyluoglu and Hickman could prove that.

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[1] A. Derbali, S. Hallara, *The Current Models of Credit Portfolio Management: A Comparative Theoretical Analysis*, "International Journal of Management and Business Research", Autumn 2012, p. 271-292.

[2] U. Koyluoglu, A. Hickman, *A generalized framework for current credit risk portfolio models*, "Risk magazine", 1998, printed 16.10.1999.

The goal is to set up a generalized framework consisting of three main components – the joint default distribution, the conditional distribution (on certain "cases" in/and sub-portfolios) and the therefrom assembled (final) unconditional default distribution. This framework allows for a detailed comparison of the industrial credit portfolio models.

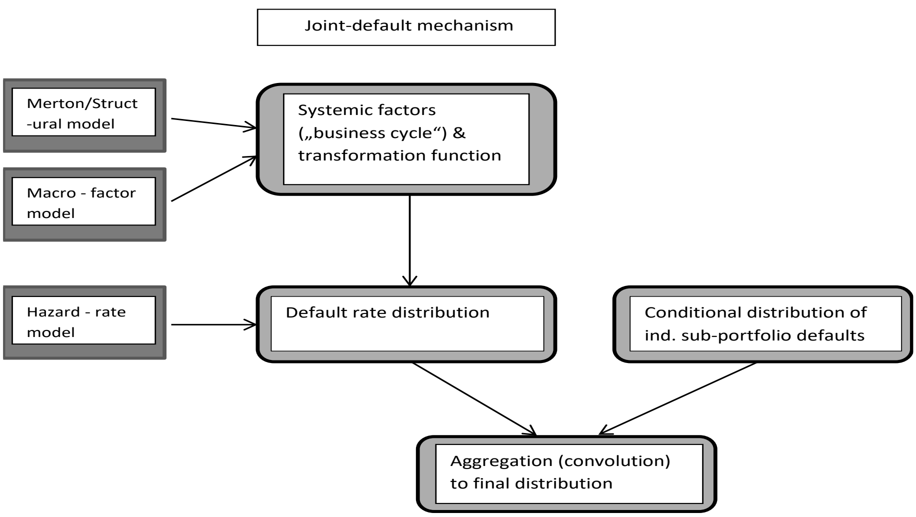
The joint default distribution is describing the correlation in the portfolio i.e. how strongly the obligors' (=debtors') conditional default rates *vary together* - in *various cases* or "states".

The second is the conditional distribution of the portfolio default rate. For each different "case" and its corresponding obligors' conditional default rates, the common conditional distribution of a homogeneous sub-portfolio default rate can be calculated as if individual defaults are independent. This homogenous sub-portfolio creation (and s.t pooling) is also the usual way for credit models and ratings when clustering different clients/debtors in a bank.

The third component is the final assembling (combination) of the data. The objective here is to achieve the unconditional distribution of portfolio defaults. This distribution is calculated straightforward by aggregating and averaging the homogeneous sub-portfolio's conditional distribution of default rate in each "state" weighted by the corresponding probability of the given state.

This – heuristically direct feasible – scheme can be represented as follows:

Fig. 1. **Scheme for deriving an aggregate final distribution out of its three components in industrial credit portfolio models**



Source: own illustration, based on A. Hickman, U. Koyluoglu, *A generalized framework for credit risk portfolio models*, "Risk magazine", 1998, p.4

The (common) default rate distribution is explicitly inherent in hazard rate models and (just) implicitly in structural (or macro-factor based) ones. Models also further link their defaults rates to variables modelling the economic cycle (known as "systemic factors") and this fact can be described by an underlying transformative conditional default rate function.

As this idea is prevalent in the models, I illustrate it in the following - oriented on Hickman: For deriving the transformative function in the structural model I decompose the change in asset value AV for asset i into a set of normally distributed *orthogonal systemic factors* x_k, and a normally distributed idiosyncratic component ϵ_i as usual in econometrics:

$$\Delta AV_i = a_{i,1}x_1 + a_{i,2}x_2 + \cdots \sqrt{1 - \sum_k a_{i,k}{}^2 \varepsilon_i} \quad (1)$$

$with\ factor\ loadings\ a_{i,k}\ and\ x_k, \varepsilon_i\ iid \sim N[0,1]$ and $\Delta AV_i\ iid \sim N[0,1]$

Hence, the described change in asset value is normally distributed with mean given by the factor loadings and factors, and standard deviation defined by the weight of the idiosyncratic factor.

In practice, the systemic factors and factor loadings are selected to exactly replicate the (empirically given) pairwise asset correlations. They are hence e.g. derived by PCA - Principal Components Analysis (or factor analysis) as usual. The assumptions for applying PCA are only very few (compared to other statistical methods), one just assumes linear relationships, continuous or ordinal variables, no outliers as far as possible and of course a sample size which is sufficient. All of these are directly incorporated in the model structure and/or reached with sufficiently available portfolio (debtors) data.

Further assumptions are just correlation-based, these are the KMO-(Kaiser-Meyer-Olkin) criterion and the Bartlett-test for spherical which are readily available in every common statistical software / programming language like SPSS, STATA, R, MATLAB or Python.

For uniqueness N (one dimensional) factors for N obligors are required - otherwise less.

Given a default threshold c, often in accordance with the debt of a company, we have the condition

$$\Delta AV_i \leq c \quad (2)$$

for defaults (as usual in structural models like CreditMetrics®).

This is obvious as when the company's asset value is (for longer) less than the debt it cannot serve its creditors anymore/cannot pay its obligations and is hence in default.
Given (2) the unconditional default probability p^* is explained by the calibration:

$$p^* = \Phi(c) \tag{3}$$

$$\textit{where } \Phi \textit{ denotes the cumulative density function } (cdf) \textit{ of the normal distribution}$$

$$p_{i|X} = \Phi\left(\frac{c - \sum_k a_{i,k} x_k}{\sqrt{1 - \sum_k a_{i,k}^2}}\right) \tag{4}$$

The default rate, conditioned on the values of systemic factors, is then (transformed; by the arguments of factor loadings defining mean and deviation).

For a homogeneous portfolio as special case, the systemic factors can be expressed by a single variable, with $\mu = \sum_k a_{i,k}{}^2$ describing the asset correlation in the homogeneous portfolio.
Then the probability density function for the default rate $f(p)$ can be – using the Radon-Nikodym-derivative - related to the probability density function of systemic factors $\phi(m)$ directly derived as:

$$f(p) = \sqrt{1-\mu}\ \frac{\Phi\left(\frac{c - \Phi^{-1}(p)\sqrt{1-\mu}}{\sqrt{\mu}}\right)}{\phi(\Phi^{-1}(p))\sqrt{\mu}} \tag{5}$$

In the hazard rate model the default rate distribution $f(p; \mu; \sigma)$ follows a Gamma distribution usually. This function empirically models the joint "enforced" behavior quite well and is therefore also used in actuarial settings. So to account for a normal distributed systemic factor the following equation for the transformation function and all points $(x; y)$ must uphold:

$$\int_0^y \Gamma(p; \alpha; \beta)\, dp = \int_x^\infty \phi(m) dm \tag{6}$$

So obviously the transformation function is given by:

$$p|m = \Theta^{-1}(1 - \Phi(m); \alpha; \beta) \tag{7}$$

$$\textit{with } \Theta \textit{ denoting the cumulative density function } (cdf) \textit{ of the Gamma function.}$$

While all models are described with respect to normally distributed systemic factors which is the distribution mainly used in the banking industry, the normal distribution is not a critical assumption- non-normality would still make they comparable in the same manner (it may just change the specific results, e.g. for more realistic fatter tails student-t or generally leptokurtic distributions can be chosen).

After describing the joint default behavior in the models, the next step as referenced above is the *conditional* distribution of portfolio default rates. Conditional means again that I consider a *homogeneous* sub-portfolio (all loans are independent given fixed i.e. conditional default rates). As usual the probability of k defaults in an n obligor-portfolio follows a Binomial distribution (default case is always a binary event: default or not).

While some models explicitly use a Binomial distribution (e.g. econometric ones) and some implicitly (like structural ones) the use of the distribution in the various models can be summed up as follows:

In structural models one calculates the change in asset value for each borrower and is then testing for default (yes/no), which is at the end equivalent to the two states yes/no of a Binomial distribution.

CreditPortfolioView® and other econometric models iteratively convoluted the individual obligor distributions – and these individual ones are all (directly) Binomial distributions.

CreditRisk+® approximates the Binomial with the Poisson distribution as described before and as the Poisson distribution is the limiting distribution for the Binomial distribution, for reasonable portfolios with small default rates (where the probability of multiple defaults is negligible) there is no significant difference as shown by Alan Stuart and Keith Ord.[3]

Hence, one can describe the Conditional Probability of default rates by a uniform Binomial setting and finally need an aggregation function as the third necessary part of the framework.

As I have described before the unconditional probability distribution of portfolio defaults is obtained by aggregating i.e. by averaging across the conditional distributions of portfolio defaults for all various "states of the world", weighted by the probability of a given state – with the help of a convolution integral.

For a homogeneous sub-portfolio with n obligors and with a single systemic factor (normally distributed) in a structural or econometric model, I therefore obtain for the convolution integral.

$$P(k\ defaults|n\ obligors) = \int_{-\infty}^{\infty} B(k;n;p|m)\phi(m)dm \quad (8)$$

The convolution integral for a homogeneous sub-portfolio in a hazard-rate model is therefore (Poisson distributed independent obligor default rates):

$$P(k\ defaults|n\ obligors) = \int_0^{\infty} P(k;np)\Gamma(p;\alpha;\beta)dp \quad (9)$$

where in (9) the convolution of the Poisson distribution and the Gamma distribution yields the Negative Binomial Distribution (abbreviated as NBD), hence a closed form solution.

[3] A. Stuart, K. Ord, *Kendall's Advanced Theory of Statistics: Volume 1: Distribution Theory*, Hodder Arnold, 6. Edition, 1994.

The integrals are directly calculable, and, in all cases, the procedures are (theoretically) exact in the limit using enough Monte Carlo simulations (in practice: structural or econometric case) respective numerical iterations and ideally small band sizes (used for hazard-rate case).

An overview of the 3-step-process I executed can be seen in the table comparing CreditMetrics®, CreditPortfolioView® and CreditRisk+®.

Table 2. **Concrete steps (3) of deriving the aggregated risk function for all three types of Credit portfolio models**

	CreditMetrics®	**CreditPortfolioView®**	**CreditRisk+®**
Joint Default-Behavior (1)	Distribution of Systemic factors (normal) | Conditional Default rate - Merton model	Distribution of Systemic factors (normal) | Conditional default rate - Macro-Regression model	Default Rate distribution (gamma distribution)
Conditional Default Distribution (2)	Binomial distribution	Binomial distribution	Poisson distribution
Aggregation function (convolution integral) (3)	Monte-Carlo simulation to evaluate the convoluted "normal-binomial"-integral	Monte-Carlo simulation to evaluate the convoluted "normal-binomial"-integral	Numeric algorithm to evaluate the convoluted "Gamma-Poisson" (=NBD) -integral

Source: own illustration.

All models rely on the parameters of conditional default probability and joint-default behavior and as seen the later parameter is the one diverging in the different models and appearing in various forms (structural models with pairwise asset correlations, the actuarial models use sector default rate volatilizes, econometric model calculates regression coefficients for macroeconomic factors incorporating correlations amongst the factors).

All these parameters for joint-default behavior are hence related and yield equivalent information for describing joint-default behavior.

To link coefficients and correlations -as described before- the joint-default behavior is represented in structural models as a pairwise asset correlation matrix, or equivalently as a set of asset factor-loadings in the following way:

$$\Delta AV_i = a_{i,1}x_1 + a_{i,2}x_2 + \cdots \sqrt{(1 - \sum_k a_{i,k}{}^2 \varepsilon_i)} \quad (10)$$

$again\ with\ factor\ loadings\ a_{i,k}\ and\ x_k, \varepsilon_i\ iid \sim N[0,1]$ and $\Delta AV_i\ iid \sim N[0,1]$

And now as systemic factors are defined to be orthonormal one can obtain:

$$Correlation[\Delta AV_i, \Delta AV_j] = \frac{E[\Delta AV_i \Delta AV_j] - E[\Delta AV_i]E[\Delta AV_j]}{\sqrt{\left(E[\Delta AV_i{}^2] - E[\Delta AV_i]^2\right) - \left(E[\Delta AV_j{}^2] - E[\Delta AV_j]^2\right)}}$$

$$= a_{i,1}a_{j,1} + a_{i,2}a_{j,2} \ldots \quad (11)$$

Hence, a pairwise correlation matrix is directly calculated given asset factor-loadings, and on the other hand factor-loadings can be derived from a pairwise correlation matrix (though the factors will not be specified).

The econometric model's logistic regression coefficients, characterizing the relationship of the default rate "index" to macroeconomic variables, obviously have a very strong similarity to the asset factor loadings of the structural models and hence an index correlation is straightforward described in the same fashion.

Now the Default rate volatility s is just calculated by the well-known standard formula for variance:

$$s^2 = \int_0^\infty (p - p^*)^2 f(p) dp \quad (12)$$

and e.g. for a structural model $(r{:}\ asset\ correlation)$:

$$s^2 = \int_0^\infty \left(\Phi\left(\frac{\Phi^{-1}(p^*) - m\sqrt{r}}{\sqrt{1-r}}\right) - p^*\right)^2 \phi(m) dm \quad (13)$$

Consequently, the default rate volatility is calculated as a function of $r\ (correlation)\ and\ p^*\ (unconditional\ default\ rate)$.

Further, for a homogeneous sub-portfolio, the variance approaches $s^2 = (1 - p^*)p^* r_{default}$ Therefore - as the infinite homogeneous portfolio's default rate volatility is precisely the parameter in the actuarial model - that yields the relationship between the default correlation and the default rate volatility (for two borrowers with the same unconditional default rate). If the default rate differs between borrowers however, a more general approach is required. This is possible using the following idea:

All models use a two - parameter default rate distribution. Hence, the mean and standard deviation (or respectively here the unconditional default rate and standard deviation of default rate) are sufficient statistics to define the parameters for any of the models.

Therefore, by setting the unconditional default rate p^* and standard deviation of default rates, one can derive the necessary parameters for all three kinds of models to describe them within the same framework which was detailed before and transform them into each other.

The structural model needs the parameters $c\ (threshold)$ and $r\ (asset\ correlation)$

With $c = \Phi^{-1}(p^*)$ and then (cf. (13) above)

$$s^2 = \int_0^{\infty} \left(\Phi\left(\frac{\Phi^{-1}(p^*) - m\sqrt{r}}{\sqrt{1-r}}\right) - p^*\right)^2 \phi(m)dm \qquad (13)$$

one can directly derive c and r by setting p^* and s.

For the econometric model I need the factor loadings and coefficients and by regrouping and setting

setting $y_{i,t} = U_i + V_i m$ (14)

where $U_i = a_{i,0} + \sum_k a_{i,k}(b_{k,0} + \sum_k a_{k,j} x_{k,t-j})$ (15)

$and\ V_i\ is\ the\ residual\ parameter$

$$V_i = \sqrt{var(U_{i,t}) + \sum_k (2a_{i,k} cov(U_{i,t}, e_{k,t}) + a_{i,k}{}^2 var(e_{k,t}) + \sum_{k<>m} a_{i,k} a_{i,m} cov(e_{m,t}, e_{k,t}))} \qquad (16)$$

$where\ m \sim N[0,1]$,

combining the index and macroeconomic variable models to a single equation for the index $y_{i,t}$ I need to derive just U and V.

This can be done with the two equations:

$$p^* = \int_{-\infty}^{\infty} \frac{1}{1 + e^{U+Vm}} \phi(m)dm \qquad (17)$$

$$s^2 = \int_{-\infty}^{\infty} \left(\frac{1}{1+e^{U+Vm}} - p^*\right)^2 \phi(m)dm \qquad (18)$$

The parameters of the actuarial model are directly derived from p^* and s by

$$\alpha = \frac{p^{*2}}{s^2}\ and\ \beta = \frac{s^2}{p^*} \qquad (19)$$

These models then imply *very similar* probability density functions for the default rate, with very small discrepancies at the tail and with extreme (low or high) default rates.[4]

Hickman and Koyluoglu conclude that “the models are virtually indistinguishable when the systemic factor is greater than negative two standard deviations, which accounts for almost 98% of the probability mass”. (p.13)

All models belong to a *single general framework*, which consists of three components

- the (joint) default rate distribution,
- the conditional default distribution,
- the convolution / aggregation use function.

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[4] U. Koyluoglu, A. Hickman, *A generalized…,* op. cit, p. 13-18.

Hence, we achieved the aim of describing them in one framework.
Any significant differences between the models "arise from differences in modeling joint-default behavior which manifest in the default rate distribution".[5] Hence after the joint default parameters are *harmonized* (by setting *p** and *s*) the default rate distributions are very similar and comparable. This fundamental result is obviously mathematically replicable and was hence confirmed by many studies like Wahrenburg and Niethen[6] (p. 14 - 20) or Schwarz[7] and of course Gordy.[8]

Significant model differences can therefore be *attributed to parameter value estimates* implicating the default rate behavior. Hence, e. g. ISDA (1998) finds that model results are fairly consistent when calibrated, whereas Roberts and Wiener (1998) conclude that the models produce very different results for same portfolios when using parameters *independently* selected according to each specific model.
H0: CreditMetrics and CreditRisk+ perform equally well (CVaR difference < 5 %):
In the independently selected case there can appear strong differences between the models with factors/multiples larger than 3, as e.g. Wahrenburg and Niethen showed with an example of n homogenous loans all from the german building sector, same sized-loans as to avoid size concentration effects on the VaR-estimator with estimators for empirical input data derived from the insolvency time series (1980-1994) of the german official federal statistics office ("Statistisches Bundesamt für das Baugewerbe") [9] as well as stock returns from building companies in the german stock index DAX 100 from that time. Gordy in another study assumes that all loans are ordinary term loans, the distribution in size and S&P rating grades is according to data from two large samples of mid-sized and large corporate loans using data from Federal Reserve Board surveys of large bank organizations. The number of obligors in the base scenario is n=5000 (with simulations for n=1000 and n=10000 as well) and the concentration calibrated by dividing the number n across the rating grades and then determining how the exposure within the grade is distributed across the number

[5] U. Koyluoglu, A. Hickman, *A generalized…,* op. cit, p. 17.
[6] M. Wahrenburg, S. Niethen, *Vergleichende Analyse alternativer Kreditrisikomodelle*, Kredit und Kapital Heft 2, 2000, 235-257. http://www.is-frankfurt.de/publikationenNeu/Vergleichende Analysealternativ654.pdf (last call: 11/13/2020, 10:21 pm), p. 14-20, summary p. 20.
[7] R. Schwarz, *Kreditrisikomodelle mit Kalibrierung der Input-Parameter,* University of Applied Sciences of bfi Vienna - Working Paper Series 3, 2004.
[8] M. B. Gordy, *A Comparative Anatomy of Credit Risk Models*, Federal Reserve System - Working Paper, 1998, p.6 - 7.
[9] M. Wahrenburg, S. Niethen, *Vergleichende Analyse…*, op. cit, p. 13.

of obligors in this grade (bracket) – using in each of the cases data from the American Society of Actuaries from 1996 yielding a sample of mid-sized and large private placement loans.

The models perform similar for average quality commercial loans portfolios when σ is small for CreditRisk+®. The author stresses the extreme sensitivity of CreditRisk+® to σ and in these cases seems to prefer CreditMetrics® as more robust. [10]

Also in other respects - especially for low default portfolios and large obligors – structural models seem to perform better (with systematically higher default rates) and hazard rate ones may underestimate default correlations as I show in the following – hence rejecting H0.

E.g. Diaz is extending the analysis mentioned carried out by Koyluoglu and Hickman (1998). As seen before, their framework allowed comparing the default distributions of both models under equivalent parameters. Diaz is extending this study by comparing CreditRisk+® and the *full version* of CreditMetrics® that considers migration risk and by setting up a slightly extended mathematical framework to compare the loss distributions. The conclusion on page 3 and page 34 -37 is that for internal purposes, CreditMetrics® is more precise and preferred[11]. The CVaR difference is up to 19%.

Just for low-quality/retail portfolios when migration risk accounts for very little in the overall CVaR (or ratings are not available) and the results between CreditMetrics® and CreditRisk+® are very similar CreditRisk+® (cf. p. 35 in Diaz paper) is a faster and less expensive approach. Two of the most comprehensive studies on comparing Credit Portfolio models an older one by Crouhy, Galai and Mark[12] and a more recent one by Boris Kollár and Barbora Gondžárová also conclude that CreditMetrics® is the preferred choice with the later study saying that the "biggest disadvantage of CreditRisk+® model comes from Poisson distribution, because it underestimates the probability of default for all rating grades" (p. 346).[13] A similar -though more broadly

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[10] M. B. Gordy, *A Comparative…*, p. 23 - 24.

[11] D. Diaz, G. Gemmill, *A Systematic Comparison of Two Approaches to Measuring Credit Risk: CreditMetrics versus CreditRisk+*, Paper at 27th ICA, International Actuarial Organisation, 2011; http://www.actuaires.org/EVENTS/congresses/Cancun/ica2002_subject/credit_risk/credit_x_diazl edezma.pdf (last call: 11/13/2020, 10:20 pm).

[12] M. Crouhy, D. Galai, R. Mark, *A comparative analysis of current credit risk models*, Elsevier, 2000.

[13] B. Kollár, B. Gondžárová, *Comparison of Current Credit Risk Models*, Paper at the 2nd GLOBAL CONFERENCE on BUSINESS, ECONOMICS, MANAGEMENT and TOURISM, 2014, p. 30-31.

formulated - concern is shared by Stein[14]. Our null hypothesis is here rejected as well.

A study by Arora, Bohn and Zhu[15] (p.19 following) confirms the preference for structural models citing "a (HW) reduced-form model largely underperforms a sophisticated structural model like that of the VK model (as implemented by MKMV)" and also confirms that the "performance of the VK model is more consistent across large and small firms, while the performance of the HW and Merton models worsens considerably across larger firms."

As mentioned before, however CreditRisk+® is easy to implement and requests less data. One can also compare the models from an *informational point of view*. Structural models assume that the modeler has *complete* information of all the obligor's assets and liabilities (i.e. as the management of the company). Hence one derives a *predictable* default time. However reduced form models assume that the modeler has just the information set *the market* has – in practice incomplete knowledge of the company's assets and liabilities. Hence the distinction between structural and reduced form models in this case is along the information set - whether the information set is observed by the market or not. Therefore [16] - at first - according to Jarrow and Protter for pricing and hedging in incomplete markets, reduced form models can be the preferred methodology. However, Giesecke and Goldberg (2004) [17]later showed that it is possible to develop a structural model in which the modeler also has incomplete information about the default point, making the time-to-default inaccessible even in a structural model. Duffie and Lando (2001)[18] propose a *hybrid model*[19] that assumes accounting information is noisy thereby making the default time inaccessible in the context of a structural model and later Jarrow in an alternative form.[20]

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[14] R. Stein, *Are the Probabilities Right? A First Approximation to the Lower Bound on the Number of Observations Required to Test for Default Rate Accuracy*, "Moody's KMV - White Paper", 2003.

[15] A. Navneet, J. R. Bohn, F. Zhu, *Reduced Form vs. Structural Models of Credit Risk: A Case Study of Three Models*, "Moody's KMV – White Paper", 2005 (and Wiley, 2012).

[16] Jarrow, R., Protter, P., *Structural versus Reduced Form Models: A New Information Based Perspective*, Cornell University-Working Paper, 2004.

[17] K. Giesecke, L. Goldberg, *Sequential Defaults and Incomplete Information*, "Journal of Risk" and "SSRN Electronic Journal", 2004.

[18] D. Duffie, D. Lando, *Term structures of credit spreads with incomplete accounting information*, econometrica, vol. 69, 2001, p. 633–664.

[19] Hybrid models are models combining structural and actuarial (hazard rate) elements. As described before CreditRisk+® is extensible by introducing multiple migration stages (instead of pure default modelling) and CreditMetrics by multi-factor modelling – yielding similar results. Cf. e. g. the works of Li.

[20] R. Jarrow, R. Cenit, *Modelling Credit Risk with Partial Information*, "The Annals of Applied Probability" 14(3), 2004.

Conclusions

Hence, Overall and in reply to the crucial question which modeling approach is better in terms of discriminating defaulters from non-defaulters and identifying relative value (in the predicted VaR compared to real portfolio data and its credit losses) CreditMetrics® is the *preferred choice as a Credit Portfolio Model in general*. Hence, the null hypothesis rejected. Especially for exchange-listed and mid-to-large size companies its probabilities of defaults and correlations are more precise than CreditRisk+® which is often underestimating the later ones. CreditRisk+® is (regarding its easy implementation and equivalent results for the segment of non-rated (scored) retail or low-quality portfolios) just preferred for *retail* portfolios.

As e.g. with specific Copula-based models (with data-fitted marginal distributions) – also multi-factor extensions of CreditMetrics® coupling all risk parameters (and their non-linear dependence structure) and hybrid models are still an interesting topic for current research and can be further very valuable for *specific tasks* and situations.

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Summary

This paper is comparing the main credit portfolio models used in the banking industry as well as in academic research for credit risk measuring and pricing purposes. These models can be classified as structural models, hazard rate models and macro-factor/ econometric models (the last group is very similar to structural ones and hence not investigated in detail here).

Proponents and famous implementations are CreditMetrics® by JP Morgan (1998) and KMV® (Kealhofer, McQuown and Vasicek, 1993) of Moody's for the structural type and CreditRisk+® by Credit Suisse First Boston (CSFB, 1997) for the hazard rate model. As all of them are frequently used the null hypothesis is that they perform equally well. In the first part of this paper – as intermediate aim - qualitative properties are compared and in the second part the models are disassembled (using a methodology similar to Hickman) and quantitatively considered with the overall goal to prove which one is measuring Credit Portfolio Risk most precisely – in contrast to existing papers not only regarding specific portfolio(s) or settings but building on a meta-analysis research of papers published on that topic. As a result – rejecting the null hypothesis - CreditMetrics® is performing better in most circumstances, especially with exchange-listed, large companies and low-default portfolios whereas CreditRisk+® is easier to handle and often useful for granular, retail type portfolios.

PORÓWNANIE MODEL ZREDUKOWANYCH I STRUKTURALNYCH W POMIARZE I ZARZĄDZANIU RYZYKIEM PORTFOLIO KREDYTOWEGO - METAANALIZA WIELOKRYTERIALNA

Streszczenie

Dokładny pomiar ryzyka kredytowego jest kluczowy z punktu widzenia instytucji finansowych. Są one zobowiązane nie tylko wypełniać wymagań prawne i zarządcze stawiane przez regulatorów takich jak EBA czy ECB w formie regulacji CRR II i CRD V

w Unii Europejskiej, ale są także zobowiązane prowadzić działalność w sposób zabezpieczający interesy biznesowe właścicieli i interesariuszy, w szczególności odnośnie do zarządzania portfolio udzielonych kredytów i związanych z nimi ryzyk (upadłości i transferu). Sprawia to, że wykorzystanie zaawansowanych modeli oceny ryzyka jest branżowym standardem.

Sytuacja w innych uwarunkowaniach jurysdykcyjnych (w Stanach Zjednoczonych czy Szwajcarii) jest podobna ze względu na uwarunkowania Basel III i wynikające z nich regulacje na poziomie krajowym. Niemniej jednak, wciąż w obszarze modelowania ryzyka kredytowego jest wiele obszarów, które nie są wystarczająco dobrze opisane. Między innymi, nie przeprowadzono metaanalizy dotychczas opublikowanych badań, która przesądzałaby nad wyższością jednych metod nad innymi. Jest to istotne, ponieważ wdrożenie skuteczniejszego modelu skutkuje przewagą konkurencyjną banku w zakresie modelowania ryzyka związanego z kredytem, a tym samym, długofalowo zmniejsza wartość zaangażowania kapitałowego niezbędnego do pokrycia ewentualnych strat. Stąd, przeprowadzenie takiej pogłębionej metaanalizy stało się celem tego artykułu. Hipoteza stanowi, że modele CreditMetrics® i CreditRisk+® są jednakowo skuteczne - w szczególności w odniesieniu do kredytów korporacyjnych. Istotną część artykułu stanowi tym samym również dokładne porównanie i wyjaśnienie mocnych oraz słabych stron modeli, w szczególności w odniesieniu do konkretnych typów portfolio czy segmentów odbiorców.