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Research Article

Application of Cramer's Rule in Economics for Determining National Income, National Consumption and Tax Revenue

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I N F O

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A B S T R A C T

The theory of matrices emerges as a valuable mathematical tool in addressing pressing economic issues, offering practical solutions to complex problems. By leveraging the power of matrix theory, a broad spectrum of challenging economic dilemmas can be effectively resolved. This study delves into the practical implementation of Cramer's rule within the realm of economics, particularly in the determination of pivotal variables such as national income, consumption patterns, and tax revenue allocation. The empirical findings underscore the efficiency of Cramer's rule, demonstrating its capacity to yield precise solutions while alleviating the burden of laborious computational tasks. This paper illuminates the significant impact of leveraging mathematical frameworks, like Cramer's rule, in streamlining economic analyses and enhancing decision-making processes. The intricate interplay between mathematical methodologies and economic analyses underscores the transformative potential of applying rigorous mathematical principles in real-world economic scenarios. Through a meticulous examination of Cramer's rule applications, this research contributes to a deeper understanding of how mathematical tools can refine economic models and facilitate strategic policy-making. Ultimately, the seamless integration of matrix theory and economic theories not only optimizes analytical outcomes but also fosters innovation and efficiency in addressing economic challenges on a broader scale.

Keywords: Matrix; Determinant; Cramer's rule; Economics; National Income; National Consumption; Tax Revenue.

Introduction:

In the fields of science and engineering, a wide range of complex issues can be effectively addressed through the utilization of matrix theory. These encompass challenges including circuit problems, characterized by intricate electrical circuitries that demand precise analysis and solutions.[1] Similarly, mechanical vibration problems, which require a deep understanding of dynamic forces and structural integrity, can be efficiently tackled through the application of matrix principles.[2] Additionally, balancing chemical reactions, an essential task in various chemical processes and industries, can be accomplished with relative ease when leveraging the versatile capabilities of matrix theory.[3, 4] By employing this mathematical framework, scientists and engineers can enhance their problem-solving abilities and streamline their workflows, leading to more efficient and accurate results in their respective fields. Overall, the integration of matrix theory into scientific and engineering practices serves as a valuable tool for overcoming technical challenges and advancing innovation in a wide array of applications.

Solving the systems of simultaneous linear equations, both homogeneous and non-homogeneous, holds significant importance in the realm of matrix theory due to their extensive applicability in various engineering and scientific fields.[5] These systems serve as fundamental tools for formulating and tackling a wide array of problems encountered in disciplines such as physics, chemistry, and computer science. In the study of matrix theory, numerous analytical and numerical methods have been developed to tackle these systems effectively. These methods play a crucial role in providing solutions to complex problems and facilitating the understanding of intricate relationships within the equations. Among the well-known techniques are Cramer's rule,[6, 7] which offers a structured approach to solving linear equations by determinants, the rank method[8, 9] for determining the rank of a matrix, and the Gauss elimination method,[10] a powerful tool for solving systems of equations through elementary row operations. Furthermore, the field of matrix theory also offers computational methods such as the Gauss-Seidel method,[10] Gauss Jordan method,[11] LU decomposition method,[11] Relaxation method,[12] and Jacobi method,[12] each tailored to address specific types of linear systems. These numerical algorithms provide efficient and accurate solutions to equations that might otherwise be challenging to solve manually. By leveraging these diverse methods, researchers and practitioners can efficiently navigate through intricate systems of linear equations and derive meaningful insights that contribute to advancements in technology and science.

The main aim of this paper is to conduct a comprehensive examination of how Cramer's rule can be effectively utilized to address various economic issues, particularly in the realms of determining national income, national consumption, and tax revenue. Through a detailed analysis of the application of Cramer's rule in the field of economics, this research seeks to shed light on the potential benefits and implications of employing this mathematical tool in solving complex problems related to economic indicators. By delving deeply into the practical implementation of Cramer's rule, the paper aims to offer insights into how this method can enhance our understanding of the intricate interplay between factors influencing national income, consumption patterns, and tax revenue generation. Through a thorough investigation and theoretical exploration, this study aims to contribute valuable knowledge to the existing body of research in the field of economics, highlighting the significance of adopting innovative mathematical approaches like Cramer's rule to address contemporary economic challenges effectively. Overall, the paper endeavors to provide a comprehensive analysis of the potential applications of Cramer's rule in the context of economic analysis, emphasizing its relevance and utility in offering new perspectives on crucial economic determinants at the national level.

Cramer's Rule for Solving the System of n Linear Non-Homogeneous Equations in n Unknowns:

Consider the following system of n linear non-homogeneous equations in n variables (unknowns) as:[8]

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots \ldots\ldots\ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots \ldots\ldots\ldots + a_{2n}x_n = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \cdots \ldots\ldots\ldots + a_{3n}x_n = b_3 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \cdots \ldots\ldots\ldots + a_{nn} = b_n \end{array}\right\} \quad (1)$$

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The system (1) can be written in matrix form as:

$$AX = B \tag{2}$$

where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots\dots\dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots\dots\dots & a_{23} \\ a_{31} & a_{32} & a_{33} & \dots\dots\dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & a_{n3} & \dots\dots\dots & a_{nn} \end{bmatrix}$

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ \vdots \\ x_n \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ \vdots \\ b_n \end{bmatrix}$

Let A be non-singular matrix. Then the solution of system (2) by Cramer's rule is given by:

$$x_i = \frac{|A_i|}{|A|}, i = 1,2,3, \dots., n \tag{3}$$

where $|A_i|$ is the determinant of the matrix A_i obtained by replacing the ith column of matrix A by the right-hand side column vector B. Now, there are three possibilities.

1. If $|A| \neq 0$ then the system (1) is consistent and has a unique solution which is given by (3).
2. If $|A| = 0$ and at least one $|A_i|$ is not equal to zero then the system (1) is inconsistent i.e., it has no solution.
3. If $|A| = 0$ and all $|A_i|$ are equal to zero then the system (1) is consistent and has infinite number of solutions.

Application of Cramer's Rule in Economics for Determining National Income, National Consumption and Tax Revenue:

If Y, C, T and t respectively represent the national income, national consumption, tax revenue and rate of tax then the following tri-regional economical model provide the information of national income, national consumption and tax revenue as:[13, 14]

$$\left.\begin{aligned} Y &= C + a \\ C &= b + c(Y - T) \\ T &= d + tY \end{aligned}\right\} \tag{4}$$

where a, b, c, d are constants.

System (4) is a system of three linear non-homogeneous equations in three unknowns Y, C, T. This system can be written as

$$\left.\begin{aligned} Y - C &= a \\ -cY + C + cT &= b \\ -tY + T &= d \end{aligned}\right\} \tag{5}$$

System (5) can be written in matrix form as:

$$AX = B \tag{6}$$

Where,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -c & 1 & c \\ -t & 0 & 1 \end{bmatrix}$$

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$$X = \begin{bmatrix} Y \\ C \\ T \end{bmatrix}$$

$$B = \begin{bmatrix} a \\ b \\ d \end{bmatrix}$$

Here $|A| = \begin{vmatrix} 1 & -1 & 0 \\ -c & 1 & c \\ -t & 0 & 1 \end{vmatrix} = 1(1-0) + 1(-c+ct) + 0(0+t)$

$= 1 - c(1-t) \neq 0$

Using Cramer's Rule, we have the required expression of national income, national consumption and tax revenue as:

$$Y = \frac{|A_1|}{|A|}, C = \frac{|A_2|}{|A|}, T = \frac{|A_3|}{|A|}$$

Now $Y = \frac{|A_1|}{|A|}$

$$= \frac{\begin{vmatrix} a & -1 & 0 \\ b & 1 & c \\ d & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & 0 \\ -c & 1 & c \\ -t & 0 & 1 \end{vmatrix}} = \frac{a(1-0) + 1(b-cd) + 0(0-d)}{1-c(1-t)}$$

$$= \frac{a+(b-cd)}{1-c(1-t)} \qquad (7)$$

Now $C = \frac{|A_2|}{|A|}$

$$= \frac{\begin{vmatrix} 1 & a & 0 \\ -c & b & c \\ -t & d & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 & 0 \\ -c & 1 & c \\ -t & 0 & 1 \end{vmatrix}} = \frac{1(b-cd) - a(-c+ct) + 0(-cd+bt)}{1-c(1-t)}$$

$$= \frac{b-cd+ac-act}{1-c(1-t)} \qquad (8)$$

Now $T = \frac{|A_3|}{|A|}$

$$= \frac{\begin{vmatrix} 1 & -1 & a \\ -c & 1 & b \\ -t & 0 & d \end{vmatrix}}{\begin{vmatrix} 1 & -1 & 0 \\ -c & 1 & c \\ -t & 0 & 1 \end{vmatrix}} = \frac{1(d-0) + 1(-cd+bt) + a(0+t)}{1-c(1-t)}$$

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$$= \frac{d-cd+bt+at}{1-c(1-t)} \qquad (9)$$

Equations (7), (8), and (9) provide the required expression of national income, national consumption, and tax revenue in terms of rate of tax.

Conclusions

Authors demonstrated their adeptness in deriving the solution to the economic model problem through the application of Cramer's rule, showcasing the effectiveness of this method. By utilizing this approach, they were able to precisely articulate the national income, national consumption, and tax revenue in relation to the tax rate. The outcomes of this study indicate that Cramer's rule offers a seamless resolution to intricate problems without the need for laborious computations. This significant finding paves the way for the potential application of this technique in various fields of science and engineering where problems are formulated using simultaneous linear equations. It is evident that the versatility and accuracy of Cramer's rule present a promising opportunity for streamlining problem-solving processes in complex systems, highlighting the importance of further exploration and utilization of this method across a broad range of disciplines. The successful demonstration of Cramer's rule in the context of the economic model problem underscores its utility as a powerful tool for addressing challenges in diverse contexts, emphasizing the importance of leveraging innovative mathematical strategies to enhance problem-solving capabilities and drive advancements in scientific and engineering applications.

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