International Journal of Applied Science and Mathematics
Volume 5, Issue 2, ISSN (Online): 2394-2894

The Importance of Topology

Jing Wang
School of Mathematics and Statistics, Shandong University of Technology, Zibo, China
Email ID: 3137975080@qq.com

Hongkui Li
School of Mathematics and Statistics, Shandong University of Technology, Zibo, China
Email ID: lhk8068@163.com

Abstract – Space is an important element of mathematical research. The study of spatial properties is a foremost task in topology. This paper discusses the importance of topology in three aspects. Firstly, taking a mathematical analysis problem as an example, we discuss the convenience of topology in solving other subject problems. Secondly, we analysis the applications of topological invariant in our real life. Finally, the spatial classification in topology makes the topology in research problems have played a transitional role.

***Keywords* – Topology, Importance, Topological Invariant, Spatial Classification.**

I. Introduction

Mathematics is a magical subject. We have different results when we define different spaces, and their scope of applications are different. Metric space solves the problem of convergence and similarity, and can prove the existence and uniqueness of solution using fixed point theorem. Normed linear space solves the problem of elemental linear operations closure, which can solve the application problem using the four major theorems (Hahn-Banach theorem, uniform bounded theorem, open mapping theorem, and closed image theorem). Sobolev space can solve the weak solution of differential equations[1]. However, with the continuous development of mathematical research, it is difficult for people to confine themselves to measuring the space to study the problem. So when people start to expand the scope of their research, there appears the discipline of topology. Topology has its own function in our daily study life.

II. The Main Results

From the math of elementary school to high school, and then the higher algebra, mathematical analysis and analytic geometry of university, the construction of these mathematical knowledge can not do without a foundation - distance. Here, distance is defined by artificial axiom and we may not define the same distance under different situations. A nonempty set plus the distance we define constitutes a metric space. At the core of metric space are different metrics, that is, different metric spaces often require different distance functions, and these different functions can describe the similarity between different elements. In metric space, different distance functions will have different clustering results, which is very helpful in studying clustering algorithms of machine learning[2]. At the same time, some problems such as sequence convergence and fixed point theorem can also be conveniently studied in metric space.

However, while using the metric space defined by distance facilitates our research, it can also hinder us in solving certain problems. For example, if the domain of a function is integer, is this function continuous? We know that in mathematical analysis, to solve this problem is difficult. Then mathematical researchers will consider to solve this problem in a different space. The following proof that this problem is very easy to solve in topological space.

In the topology, instead of defining distance, we redefine open sets, neighborhoods, closed sets, closures and interior directly using the set as a basis and define the topological space in a variety of ways. In topological space, we use open sets to describe continuous mappings: Let X and Y be two topological spaces. $f: X \to Y$. If the preimage of each open set U in Y is an open set in X, then f is a continuous mapping from X to Y. Then find a special topological space - discrete topological space: Suppose M is a set. Let $^{2} = \mathrm{P}(M)$, that is, a family of all subsets of M. Then ^{2} is a topology of M,, which is called a discrete topology, and we call the topology space $(M, ^{2})$ is a discrete space[3]. The characteristic of discrete topological space is that every subset of it is open sets. With the above bedding, we can easily get:

1) Let integer set Z be a subset of the real number space $(R, \ )$. We define the topology of Z that is $_Z = \{Z \cap Y | Y \in \ _Z\}$. Then $(Z, \ _Z)$ is a subspace of topology space $(R, \ )$.

Arbitrarily select $\mathrm{n} \in Z$ to satisfy

$$\{\mathrm{n}\} = Z \cap (\mathrm{n} - \frac{1}{2}, n + \frac{1}{2}) .$$

$(\mathrm{n} - \frac{1}{2}, n + \frac{1}{2})$ is an open set in R, so $\{\mathrm{n}\} \in \ _Z$.

That is, the set of single points in Z is an open set. From the property of the open set, we can see that any union of these one-point sets is also an open set. So any subset of Z is open set. Thus Z is a discrete space as one of subspace of R.

2) For a mapping f from a discrete space M to a topological space X. M is a discrete space, so any subsets of M are open sets. And for $\forall A \subset X$, there is $f^{-1}(A) \subset M$. So $f^{-1}(A)$ is a open set of M. Thus f is continuous. So any mapping from discrete space to topological space is continuous mapping.

From1) and 2) shows that if a function domain is integer space, then this function is continuous mapping.

So the problem of mathematical analysis that we just mentioned is solved in the topology.

As another example, when we solve a problem, we often expand the function to the result we want, and it is a problem to judge whether this function can be expanded. In topology, we study the Tietze expansion theorem: Let X be a topological space and $[a,b]$ be a closed interval. Then X is a normal space if and only if for any a closed set A in X and any a continuous mapping

$$f : A \to [a,b]$$

there is a continuous map $g : X \to [a,b]$ is the expansion of f [3]. According to this theorem we can determine whether a function can be expanded. For example, a function $f(x) = \sin\left(\frac{1}{x}\right)$ can not be expanded to a continuous map on $X \to [a,b]$.

Of course, this is just a small example of using topology to solve mathematical analysis problems. Topology has many applications in other branches of mathematics and other subjects. In other disciplines of mathematics, the application of topological simple hematology theory to the formation of polyhedron embodies the process of algebraic transformation of geometric problems[4]; In chemistry and chemical industry, we introduce the concept of topological index and we use topological chemical reaction control principle in synthetic chemistry and industrial production [5]; In philosophy, the study of topology has a great fit with the nature of philosophical research. First, the philosophical concepts discussed in the conceptual space are time-dependent. Philosophy is not concerned with the special properties of these rheologies, but with the general nature of the conceptual space that holds these rheologies. This has in common with the topological study of the concept of space; Second, philosophy emphasizes the continuity and universality of concepts. The persistence of concept and the permanent life of thought in different historical development have always been the goals of philosophical research. This is also consistent with the goal of topology research; third, if conceptual research is to be regarded as an important part of philosophical research, then the conceptual space consisting of concepts prescribes the formation and change of philosophical concepts. This can also be understood from the topological research ideas[6].

The most important aspect of topology is studying how different space in a continuous mapping to maintain the same amount. In real life, the topological invariant is the property of keeping the object constant after continual deformation. That is, a line no matter how it is still a line as long as no tie knot, and a dough no matter how knead it still a dough just do not tearing it up. Euler in the settlement of konigsberg bridge problem, when he painted the graph without considering its size and shape is the use of this principle. At the same time it is also a great application in architectural design. Topology in graph theory, knot theory can be applied directly to the architectural design modeling. Connectivity, topology embedding and other characteristics are infiltrated into the building space. This has become the theoretical basis for the development of space[7]. Therefore, when discussing the properties of connectedness, axiom of countable and separability, we discuss whether these properties are invariant under continuous mapping. And then study the topological invariant properties of geometric shapes, so as to linking with the actual study of different space in a continuous change in the amount of unchanged.

From another perspective, in the view of space, topology discusses the problem of classifying space. For example, one-dimensional Euclidean space and two-dimensional Euclidean space is not the same as a one-dimensional removal of a point into two non-connected intervals, and two-dimensional removal of a point is still a connected area. The difference between a two-dimensional European space and a three-dimensional European space is that one point is removed in two dimensions and the rubber band around this point is reduced by less than one point, but three-dimensional is possible. By different topological properties to distinguish between different spaces, so as to achieve the results we need. At the same time we can also build a fabric model based on this principle. Among them, the fiber can be expressed in a set of real numbers. Due to different perspectives, we can divide it into fiber structure space and fiber space. Both spaces have corresponding measures to measure the gap between any two different fibers in space[8]. In particular, we are in the study of topological space at the same time it is slowly moving toward the metric space or n-dimensional European space closer. Thus appeared Hausdorff space. Only the transition of metric space to topological space and the transition of topological space to metric space can play the role of topological space in solving the problem of metric space.

III. CONCLUSION

In any case, topological space is very useful for the study of mathematical problems. The application of topological space in other disciplines and the connection of other disciplines in topological studies still need further study. This will be our research direction.

ACKNOWLEDGMENT

This work is supported by Doctoral Fund of Shandong University of Technology of China (4041/415061), Comprehensive budget management system design (9101/215043), Shandong Research project of teaching reform in undergraduate colleges and universities of China (2015Z071).

This work is supported by Open Project Foundation of Intelligent Information Processing Key Laboratory of Shanxi Province (2017003).

REFERENCES

[1] Zhang Shisheng. The basic theory of probability measurement space and its application (I) [J]. Applied Mathematics and Mechanics, 1988 (02): 117-126.

[2] Niu Fenggao, Zhang Rongjie. Text clustering evaluation method based on intra-class distance parameter estimation[J]. Journal of Shanxi University (Natural Science Edition): 2009, 35(9):37-39. .

[3] XIONG Jincheng. Point Set Topology Notes [M]. 3rd ed Beijing: Higher Education Press, 2013, pp. 57–135.

[4] Wang Rulu, Zhang Jiasheng, Liu Baochen. New Application of Topology - Geometric Recognition of 3D Blocks [J].Mining and Metallurgical Engineering, 1997 (03): 19-22 + 25.

[5] SHI Hai-Xin, HUANG Dong-Mei, TAN Ming-Ji, HONG Chun-Mei.Application of Topology Theory in Chemistry and Chemical Engineering [J]. Chemical Engineer, 2010,24 (07): 38-41.

[6] Jiang Yi. How to Understand the Nature of Philosophy from Topology [J].Journal of the Chinese Academy of Social Sciences, 2010 (03): 32-37.

[7] Wu Po. On the topology in the application of architectural design [D]. Tianjin University, 2012.

[8] Wang Rulu, Zhang Jiasheng, Liu Baochen. New Application of Topology - Geometric Recognition of 3D Blocks [J].Mining and Metallurgical Engineering, 1997 (03): 19-22 + 25.

AUTHOR'S PROFILE

Wang Jing, Shandong in China, was born in 1997 and studying at the third grade of Shandong University of Technology now. She is now a third-year student at Shandong University of Technology. She has published a dissertation in the International Journal of Trend in Research and Development.
Ms. Wang won the M Prize at the 2017 The Mathematical Contest in Modeling and winning the second prize in the 2017 Asia Pacific Mathematical Contest for College Students, the Eighth National Undergraduate Mathematics Contest and the Seventh Shandong College Students Mathematical Contest.

Corresponding Author:**Hongkui Li**