# DESIGN OF THE CONSTANTINESCO TORQUE CONVERTER USED AS A MECHANICAL TRANSMISSION FOR AGRICULTURAL TRACTORS 

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# DIMENSIONNEMENT DU CONVERTISSEUR DE COUPLE DE CONSTANTINESCO UTILISÉ COMME TRANSMISSION MÉCANIQUE DES TRACTEURS AGRICOLES 

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#### Abstract

The Constantinesco torque converter (CTC) is a continuously variable mechanical transmission and was introduced by the eminent engineer George Constantinesco during the first half of the twentieth century. This article provides a brief historical overview of the development of the CTC as well as its general functioning principle. A procedure for the integration of the equations of motion of the CTC using the structomatic approach is proposed for the purpose of its sizing. The general dynamic behaviour of the CTC is investigated. The results emphasize the potential of incorporating the CTC into agriculture.


## RÉSUMÉ

Le convertisseur de couple de Constantinesco (CCC) est une transmission mécanique à ratios variables proposée par l'éminent ingénieur George Constantinesco durant la première moitié du vingtième siècle. Cet article présente un aperçu historique du développement du CCC ainsi que son principe général de fonctionnement. Une procédure pour l'intégration des équations de mouvement du CCC utilisant les principes de la structomatique des mécanismes est proposée dans le but de procéder à son dimensionnement. Le comportement dynamique général du convertisseur de couple de Constantinesco est investigué. Les résultats mettent l'emphase sur le potentiel d'incorporer le CCC dans le domaine agricole.

## INTRODUCTION

Agricultural tractors are frequently used in agriculture to tow equipment such as manure spreaders, trailed sprayers and harvesting equipment or to support certain agricultural implements such as forks or front-wheel-drive rollers and plows at the front or dethatchers at the back. These machines are characterized by a high inertia (empty weight of several tonnes) and thus requires considerable power when they start to move from rest. Several decades ago, the current manufacturers (John Deere, CNH Industrial, Kubota, etc.) have introduced continuously variable transmissions (CVT) to adapt the torque developed to the wheels according to the speed of the tractor, and this, to deal with wide scenarios of solicitation. The proposed transmission models (Fendt Vario, Cnh, ZF, Valtra, etc.) usually include two main parts: the first is hydraulic and the second is mechanical. The use of an epicyclic train is the basis for the operation of these transmissions.

This paper investigates an alternative to the actual CVTs. The Constantinesco torque converter (CTC), which was proposed by the eminent engineer George Constantinesco in the first half of the twentieth century, is a purely mechanical CVT consisting only of links. One of the particularities of this torque converter is that it omits the use of gear trains, clutches, differentials and other components frequently encountered in mechanical systems thus reducing the weight as well as the maintenance time of conventional gearboxes.

During the 1924 Wembley and 1926 Paris exhibitions, the CTC attracted attention not only for its ingenuity and the energy savings it advocated but also for the scientific advances it brought. At these exhibitions, the CTC was mounted on a vehicle (Constantinesco model) and, during the public displays, had shown positive results and it even ensued from discussions with General Motors. Several details concerning the interest and development of the CTC can be consulted in the book written by (Constantinesco, l., 1994).

To explain the general operating principle of the Constantinesco torque converter, the model of Fig. 1 is used. This model can be modified, for example, by substituting the mass at the end of the pendulum 3 by a flywheel replacing the element 4. Other models are presented in Constantinesco's patents or in his treatise
on Sonics (Constantinescu, G., Marinescu, M., Petcu, M.,Jianu, C., 1985). However, he recommended that the configuration of Fig. 1 was the most stable.

Briefly, the CTC consists of three subsystems: motor (crank 1 and connecting rod 2), accumulator (double pendulum 3 and 4) and rectification (elements 5, 6, 7 and 8 ). The motor subsystem induces harmonic motion to the joint $C$ which leads to an oscillating motion of the accumulator subsystem. The rectification subsystem converts the reciprocating motion of the joint $D$ into a unique direction of rotation due to the one way bearings that are mounted on the output shaft H . Constantinesco had named this subsystem mechanical diode by analogy with electricity. The originality of the CTC comes from the fact that the pendulum 3, element of greater inertia, is able to store the mechanical energy developed by the motor at A when the resistance at the output shaft H can not be overcome or simply varied. Thus, after a certain accumulation time, the CTC is able to ensure the motion at the output shaft while using a motor of lower rated power. In addition, the system equipped with the CTC does not stop or block when the resistance increases suddenly and drastically at the output shaft. Constantinesco exposed more details about the functioning of his invention in his paper of 1926 (Constantinesco, G., 1926). A notable paper of the early twentieth century is the one of (Jack, R.K., 1927). (McNeil, I., 1982) also treated of the functioning principles of the CTC in his work.


Fig. 1 - Theoritical model of the Constantinesco torque converter
The Constantinesco torque converter has two degrees of mobility whose generalized coordinates are $q_{1}$ and $q_{2}$ in Fig. 1, but is actuated by a single motor located in $A$. For their part, the angles $\varphi_{i}$ are the dependent coordinates. The second degree of mobility is relative to its ability to accumulate mechanical energy. Its dynamic behavior is sensitive to the variation of several parameters, mainly the inertia of the pendulum 3, the lever arm between the joint $D$ and the mass of the pendulum 3, the location of the fixed joints $F$ and $H$ and the radius of the crank 1 . The study of the stability of the CTC is a subject that can not be neglected. Moreover, a dynamic model of the CTC has been proposed by (lon-Guta, I., 2016) based on the Lagrange multiplier method. He also discusses stability by presenting Bode diagrams when the motor speed is constant and the output resistance profile is predetermined.

In the next sections, the dynamic model will be constructed from Lagrange's equations. The numerical procedure for the integration of equations of motion will be presented. The distinctiveness of this procedure is related to the use of the notions of structomatics of mechanisms to compute the transmission matrices. Subsequently, the dynamic results of some simulations will be discussed.

The design of any system goes through its dynamic analysis. This allows first to deduce the kinematic parameters of all the elements and to know the reactions acting on each of the joints. Ultimately, from these reactions, the stresses soliciting each of the elements are deduced and we can proceed to the sizing. The equations of motion are also prerequisites to the study of the dynamic stability of the system in question.

In this paper, the motion of the Constantinesco torque converter will be studied in the situation where the angular velocity of the motor of Fig. 1 is constant. Thus, it is the driving torque required to ensure that this motion law is respected that is calculated. This approach is called inverse dynamics.

Before discussing, in the next section, the dynamic behaviour of the CTC, let us introduce the dynamic procedure put forward to integrate the equations of motion of any linkage mechanisms. It comes down to the following steps:

1) Deduce the sequence of transmission of motion of the mechanism by using the principles of structomatics of mechanisms;
2) Calculate the transmission matrices of velocity and acceleration from the kinematic models of the structomats;
3) Calculate the vector of generalized forces;
4) Integrate the equations of motion with a pre-established numerical method;
5) Save the results.

The sequence of transmission of motion is essential to implement the proposed dynamic procedure. In fact, the general principle of the formation of mechanisms states that any mechanism can be partitioned into a succession of unique groupings (structomats) connected in series or in parallel (Duca, C., Simionescu, I., 1973). In this way, instead of studying the mechanism as a whole from the outset, it is split into several groupings which have the same dynamic behaviour as the starting mechanism. Each of these unique groupings or structomats is associated with a kinematic and kinetostatic model that can be implemented on a computer software such as Matlab in order to build a library. This library can be easily reused to study a wide range of mechanisms. Since the kinematics of the mechanisms is nonlinear, partitioning the mechanism into a succession of structomats makes it possible to simultaneously solve a smaller number of nonlinear equations.

Regarding the Constantinesco torque converter, its movement is dictated by the connection between two driving elements $R$ (elements 1 and 4) and three dyads RRR formed of the pairs of elements 2 and 3, 5 and 6 as well as 7 and 8 . In the notation used, $R$ means a rotary joint. This subdivision is shown in Fig. 2a. The sequence of transmission of motion is illustrated using the block diagram of Fig. 2 b .


Fig. 2 - a) Highlighting the structomats constituting the CTC; b) Block diagram of the CTC
The kinematic and kinetostatic models of the driving element $R$ and the dyad RRR are shown in Fig. 3 and Fig. 4 respectively. As an example of kinematic modelling, (Moise, V., Tabara, I., Dugaesescu, I., Dudici, C.L., Niculae, E., Rotaru, A.,Polena, A., 2017) presented the detailed procedure related to the kinematics of the driving hexade. To perform the kinematic analysis, the subroutines associated with the structomats are called in the order prescribed by the block diagram, that is to say from the zeropole to the final structomats while the kinetostatic runs in the opposite direction. The zeropole is the base or chassis and is always the first structomat to appear in the sequence of transmission of motion.

The driving element $R$ of Fig. 3a is an active structomat whose kinematic parameters of joint $A$ and element 1 are imposed and thus known. The kinematic model of the driving element $R$ makes it possible to calculate the kinematic parameters of any points of interest such as the center of gravity or other joints connected to the element 1. The dyad RRR of Fig. 3b is a passive structomat where, in order to be able to calculate the kinematic parameters of its elements and joints, those of joints $A$ and $C$ must be previously calculated.

a)

b)

Fig. 3 - a) Kinematic model of the driving element $R \quad$ b) Kinematic model of the dyad RRR

a)

b)

Fig. 4 - a) Kinetostatic model of the driving element R ; b) Kinetostatic model of the dyad RRR

In Fig. 3, the angles $q_{1}, \varphi_{1}$ and $\varphi_{2}$ are shown for reference purposes and are used to build the subroutines in Matlab. When the whole mechanism is studied, we relate those reference angles to the angles describing its real motion. The following paragraphs describe, in a general way, the proposed numerical procedure to write the equations of motion of a mechanism with $M$ degrees of mobility.

Analytically, the dependent coordinates $\varphi_{i}$ of a mechanism can be written as a function of the $M$ generalized coordinates that are describing its entire motion:

$$
\begin{equation*}
\varphi_{i}=\varphi_{i}\left(q_{1}, q_{2}, \ldots, q_{M}\right) \tag{1}
\end{equation*}
$$

Derivation of equation (1) as a function of time, the expressions of the dependant velocities as well as the dependent accelerations can be written as:

$$
\begin{gather*}
\dot{\varphi}_{i}=\sum_{j=1}^{M} \frac{\partial \varphi_{i}}{\partial q_{j}} \cdot \dot{q}_{j}  \tag{2}\\
\ddot{\varphi}_{i}=\sum_{j=1}^{M} \frac{\partial \varphi_{i}}{\partial q_{j}} \cdot \ddot{q}_{j}+\sum_{j=1}^{M} \sum_{k=1}^{M} \frac{\partial^{2} \varphi_{i}}{\partial q_{j} \partial q_{k}} \cdot \dot{q}_{j} \dot{q}_{k} \tag{3}
\end{gather*}
$$

where $\partial \varphi_{i} / \partial q_{j}$ et $\partial^{2} \varphi_{i} / \partial q_{j}^{2}$ are respectively the transmission functions of velocity and acceleration. These functions are also called influence coefficients in the literature. These transmission functions are essential for writing the equations of motion in a matrix formulation. Extracting the analytical expressions of these transmission functions can be mathematically cumbersome. This is why a numerical approach is recommended.

To illustrate the importance of using the kinematic models of the structomats, we start by writing down the equations of position of the dyad RRR of Fig. 3b :

$$
\begin{align*}
& x_{A}+A B \cos \varphi_{1}-B C \cos \varphi_{2}-x_{C}=0  \tag{4}\\
& y_{A}+A B \sin \varphi_{1}-B C \sin \varphi_{2}-y_{C}=0
\end{align*}
$$

which is a nonlinear system of equations that is solved with a iterative method such as the one of NewtonRaphson. In this way, before going forward with the velocity calculation, the angles $\varphi_{1}$ and $\varphi_{2}$ are calculated. Deriving the system of equations of position (4) as a function of time, we obtain the linear system of equations of velocity:

$$
\begin{align*}
& \dot{x}_{A}-A B \sin \varphi_{1} \cdot \dot{\varphi}_{1}+B C \sin \varphi_{2} \cdot \dot{\varphi}_{2}-\dot{x}_{C}=0  \tag{5}\\
& \dot{y}_{A}+A B \cos \varphi_{1} \cdot \dot{\varphi}_{1}-B C \cos \varphi_{2} \cdot \dot{\varphi}_{2}-\dot{y}_{C}=0
\end{align*}
$$

By imposing successively in (5) that all the generalized velocities are zero except for $\dot{q}_{j}=1$, the expression (2) of the dependent velocities are simplified to $\dot{\varphi}_{1}=\partial \varphi_{1} / \partial q_{j}$ and $\dot{\varphi}_{2}=\partial \varphi_{2} / \partial q_{j}$. The only resulting unknowns of the system of equations of velocity (5) are therefore these same transmission functions.

The transmission functions of acceleration are calculated from the system of linear equations of acceleration obtained by deriving the system (5) as a function of time and by replacing the dependent accelerations by their expression given in (3).

$$
\begin{align*}
& \ddot{x}_{A}-A B \sin \varphi_{1} \cdot \ddot{\varphi}_{1}-A B \cos \varphi_{1} \cdot \dot{\varphi}_{1}^{2}+B C \sin \varphi_{2} \cdot \ddot{\varphi}_{2}+B C \cos \varphi_{2} \cdot \dot{\varphi}_{2}^{2}-\ddot{x}_{C}=0 \\
& \ddot{y}_{A}+A B \cos \varphi_{1} \cdot \ddot{\varphi}_{1}-A B \sin \varphi_{1} \cdot \dot{\varphi}_{1}^{2}-B C \cos \varphi_{2} \cdot \ddot{\varphi}_{2}+B C \sin \varphi_{2} \cdot \dot{\varphi}_{2}^{2}-\ddot{y}_{C}=0 \tag{6}
\end{align*}
$$

Considering that all generalized accelerations are zero, the transmission functions of acceleration $\partial^{2} \varphi_{i} / \partial q_{j}^{2}$ are obtained first when the set of generalized velocities are zero except for $\dot{q}_{j}=1$ and the mixed transmission functions of acceleration $\partial^{2} \varphi_{i} / \partial q_{i} \partial q_{j}$ are deduced afterwards when the pair $\dot{q}_{i}$ and $\dot{q}_{j}$ of generalized velocities have simultaneously a unit value when the others are zero. In summary, the transmission functions are calculated by performing the kinematic analysis of the mechanism for particular values of the generalized velocities. An example of kinematic analysis using the structomatic approach is given by (Mailloux, M., Éné, M., Simionescu, I.,Tabara, I., 2017).

This calculation of the transmission functions is relative to a geometrical approach carried out from the closing equations on the contour of the open kinematic chains that are the structomats. To make the bond between structomat and transmission functions, the dyad of Fig. 3b has been used, but this approach can be applied to any type of structomats.

From a dynamic point of view, we are interested by the Lagrange's approach. In this way, the expression of the kinetic energy is needed to develop the equations of motion. Details on the development of equations of motion by using the following method can be found in the work of (Manolescu, N.,Dranga, M., 1975) and (Ene, M., 2013).

In this context, the kinematic parameters of the centers of gravity of each of the elements of the mechanism are included in the elementary transmission vector of position:

$$
\mathbf{P}_{i}=\left[\begin{array}{lll}
x_{G_{i}} & y_{G_{i}} & \varphi_{i} \tag{7}
\end{array}\right]^{T}
$$

where $G_{i}$ is the center of gravity of the $i$-th element of the mechanism.
When the vector (7) is derived as a function of time, it yields to the components of the velocity vector at the center of gravity as well as the angular velocity of the element:

$$
\mathbf{v}_{i}=\frac{d \mathbf{P}_{i}}{d t}=\left[\begin{array}{lll}
v_{i x} & v_{i y} & \omega_{i}
\end{array}\right]^{T}=\mathbf{V}_{i} \cdot \dot{\mathbf{q}}=\left[\begin{array}{cccc}
\frac{\partial x_{G_{i}}}{\partial q_{1}} & \frac{\partial x_{G_{i}}}{\partial q_{2}} & \cdots & \frac{\partial x_{G_{i}}}{\partial q_{M}}  \tag{8}\\
\frac{\partial y_{G_{i}}}{\partial q_{1}} & \frac{\partial y_{G_{i}}}{\partial q_{2}} & \cdots & \frac{\partial y_{G_{i}}}{\partial q_{M}} \\
\frac{\partial \varphi_{i}}{\partial q_{1}} & \frac{\partial \varphi_{i}}{\partial q_{2}} & \cdots & \frac{\partial \varphi_{i}}{\partial q_{M}}
\end{array}\right] \cdot\left[\begin{array}{c}
\dot{q}_{1} \\
\dot{q}_{2} \\
\vdots \\
\dot{q}_{M}
\end{array}\right]
$$

where $\mathbf{V}_{i}$ is the elementary transmission matrix of velocity which contains the transmission functions of velocity $\partial \varphi_{i} / \partial q_{j}$.

For a mechanism consisting of $m$ moving elements, the transmission matrix of velocity takes the form:

$$
\mathbf{V}=\left[\begin{array}{llll}
\mathbf{V}_{1}^{T} & \mathbf{V}_{2}^{T} & \cdots & \mathbf{V}_{m}^{T} \tag{9}
\end{array}\right]^{T}
$$

so that the kinetic energy of the entire mechanism can be written in matrix form as follows:

$$
\begin{equation*}
T=\frac{1}{2} \dot{\mathbf{q}}^{T} \mathbf{V}^{T} \mathbf{M} \mathbf{V} \dot{\mathbf{q}} \tag{10}
\end{equation*}
$$

where $\mathbf{M}=\operatorname{diag}\left(\begin{array}{llll}\mathbf{M}_{1} & \mathbf{M}_{2} & \cdots & \mathbf{M}_{m}\end{array}\right)$ is the matrix of inertia that incorporates mass $m_{i}$ and the moment of inertia $J_{G_{i}}$ of each elements :

$$
\mathbf{M}_{i}=\left[\begin{array}{ccc}
m_{i} & 0 & 0  \tag{11}\\
0 & m_{i} & 0 \\
0 & 0 & J_{G_{i}}
\end{array}\right]
$$

The components of the acceleration at the center of gravity of the $i$-th element correspond to the second derivative of the elementary position vector (7) :

$$
\mathbf{a}_{i}=\left[\begin{array}{lll}
a_{i x} & a_{i y} & \alpha_{i} \tag{12}
\end{array}\right]^{T}=\frac{d^{2} \mathbf{P}_{i}}{d t^{2}}=\mathbf{V}_{i} \cdot \ddot{\mathbf{q}}+\frac{d \mathbf{V}_{i}}{d t} \cdot \dot{\mathbf{q}}=\mathbf{V}_{i} \cdot \ddot{\mathbf{q}}+\mathbf{A}_{i} \cdot \mathbf{E} \cdot \dot{\mathbf{q}}
$$

where the generalized acceleration vector is $\ddot{\mathbf{q}}=\left[\begin{array}{llll}\ddot{q}_{1} & \ddot{q}_{2} & \cdots & \ddot{q}_{M}\end{array}\right]^{T}$ and $\mathbf{A}_{i}=\left[\begin{array}{lllll}\mathbf{A}_{i}^{1} & \mathbf{A}_{i}^{2} & \cdots & \mathbf{A}_{i}^{M}\end{array}\right]$ is the elementary transmission matrix of acceleration with :

$$
\mathbf{A}_{i}^{j}=\left[\begin{array}{cccc}
\frac{\partial^{2} x_{G_{i}}}{\partial q_{j} \partial q_{1}} & \frac{\partial^{2} x_{G_{i}}}{\partial q_{j} \partial q_{2}} & \cdots & \frac{\partial^{2} x_{G_{i}}}{\partial q_{j} \partial q_{M}}  \tag{13}\\
\frac{\partial^{2} y_{G_{i}}}{\partial q_{j} \partial q_{1}} & \frac{\partial^{2} y_{G_{i}}}{\partial q_{j} \partial q_{2}} & \cdots & \frac{\partial^{2} y_{G_{i}}}{\partial q_{j} \partial q_{M}} \\
\frac{\partial^{2} \varphi_{i}}{\partial q_{j} \partial q_{1}} & \frac{\partial^{2} \varphi_{i}}{\partial q_{j} \partial q_{2}} & \cdots & \frac{\partial^{2} \varphi_{i}}{\partial q_{j} \partial q_{M}}
\end{array}\right]
$$

The matrix $\mathbf{E}$ encloses M times the vector of generalized velocities on its diagonal.
As for the transmission matrix of velocity (9), the transmission matrix of acceleration comprises all the elementary transmission matrices of acceleration such that:

$$
\mathbf{A}=\left[\begin{array}{llll}
\mathbf{A}_{1}^{T} & \mathbf{A}_{2}^{T} & \cdots & \mathbf{A}_{m}^{T} \tag{14}
\end{array}\right]^{T}
$$

The kinematic parameters of the center of gravity of each of the elements are obtained by using the kinematic model of the driving element following the kinematic analysis of each of the structomats. As it was the case for the transmission functions of velocity and acceleration, the partial derivatives included in the transmission matrices of velocity $\mathbf{V}$ and of acceleration $\mathbf{A}$ are obtained by imposing particular values on generalized velocities and accelerations. For instance, imposing only $\dot{q}_{j}=1$ when the other generalized velocities are zero, the components $v_{i x}$ and $v_{i y}$ of the center of gravity of the $i$-th element correspond to the partial derivatives $\partial x_{G_{i}} / \partial q_{j}$ and $\partial y_{G_{i}} / \partial q_{j}$ respectively.

A peculiarity of the proposed procedure originate from the fact that the equations of motion are written in matrix form as follows:

$$
\begin{equation*}
\mathbf{V}^{T} \mathbf{M V} \ddot{\mathbf{q}}+\mathbf{V}^{T} \mathbf{M A E} \dot{\mathbf{q}}=\mathbf{V}^{T} \mathbf{F} \tag{15}
\end{equation*}
$$

obtained by inserting the expression (9) of the kinetic energy of the mechanism into the Lagrange equations:

$$
\begin{equation*}
\frac{d}{d t}\left(\frac{\partial T}{\partial \dot{q}_{j}}\right)-\frac{\partial T}{\partial q_{j}}=Q_{j}, \quad j=1,2, \ldots, M \tag{16}
\end{equation*}
$$

The vector $\mathbf{F}=\left[\begin{array}{llll}\mathbf{F}_{1}^{T} & \mathbf{F}_{2}^{T} & \cdots & \mathbf{F}_{m}^{T}\end{array}\right]$ is that of the generalized forces where the elementary force vector of the $i$-th element of the mechanism includes the external forces soliciting it such that:

$$
\mathbf{F}_{i}=\left[\begin{array}{lll}
F_{i x} & F_{i y} & M_{i} \tag{17}
\end{array}\right]^{T}
$$

where the weight of the element is considered as an external force and is included in the component $F_{i j}$.
The numerical method used for the integration of the equations of motion is the fourth order predictorcorrector method of Adams-Moulton coupled with the one of fourth order Runge-Kutta for the first approximations:

$$
\begin{align*}
& y_{n+1}^{p}=y_{n}+\frac{h}{24}\left(55 f_{n}-59 f_{n-1}+37 f_{n-2}-9 f_{n-3}\right)  \tag{18}\\
& y_{n+1}=y_{n}+\frac{h}{24}\left(9 f_{n+1}^{p}+19 f_{n}-5 f_{n-1}+f_{n-2}\right)
\end{align*}
$$

Adams-Moulton's multiple step method is prioritized because it has a precision comparable to that of Runge-Kutta but requires only two evaluations of the equations of motion at each instant compared to four for the latter. To be able to be implemented, the equations of motion (15) are transformed into a system of first-order differential equations in the form of $\dot{\mathbf{y}}=\mathbf{f}(t, \mathbf{q}, \dot{\mathbf{q}})$. For each steps, it is necessary to update the values of the transmission matrices.

A special feature related to the inverse dynamics of a mechanism is that, by imposing a constant angular velocity $\dot{q}_{1}$, the angular displacement follows $q_{1}=\left(q_{1}\right)_{0}+\dot{q}_{1} t$ whereas the angular acceleration $\ddot{q}_{1}$ is zero. Here, $\left(q_{1}\right)_{0}$ is an initial condition. In this case, it is the motor torque $M_{1}$ which corresponds to the unknown so that $M-1$ equations of motions of (15) are solved with the numerical method (18) before substituting the kinematic parameters obtained in the remaining equation of motion to deduce the resulting motor torque.

When all the kinematic parameters of the joints and the elements of a mechanism are known, the kinetostatic models of the structomats can be used to deduce the reactions at the joints. Those of the driving element $R$ and the dyad RRR of the Constantinesco torque converter are shown in Fig. 4.

Then, the stresses are calculated in order to proceed to the selection of the engine(s), the material of the elements, the bearings, etc. Wear and fatigue analyses can also be conducted.

## RESULTS

This article presents a preliminary study of the dynamic behaviour of the Constantinesco torque converter of Fig. 1. Thus, the constructive data used for the dynamic study are not those that advocate an optimal configuration, that is to say the one where the efficiency is maximized.

Table 1 groups together the lengths of each of the elements as well as their mass used for the simulations. Each of the elements are considered to be thin rods where the center of gravity is located at their midpoint. The initial conditions needed to solve nonlinear position equations are also included.

Table 1
Constructive data for the dynamic simulations of the CTC

| Length (m) |  | Mass (kg) |  | Initial conditions $\left.{ }^{\circ}{ }^{\circ}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A B=0.02$ | $E F=0.15$ | $m_{1}=0.74$ | $m_{6}=2.73$ | $\left(\varphi_{1}\right)_{0}=122$ | $\left(\varphi_{5}\right)_{0}=315$ |
| $B C=0.45$ | $E M=0.60$ | $m_{2}=16 ., 40$ | $m_{7}=2.73$ | $\left(q_{2}\right)_{0}=265$ | $\left(\varphi_{6}\right)_{0}=214$ |
| $C E=0.45$ | $G H=0.30$ | $m_{3 \_1}=21.87$ | $m_{8}=2.73$ | $\left(\varphi_{1}\right)_{0}=350$ |  |
| $D E=0.25$ | $H I=0.30$ | $m_{3 \_2}=12.00$ |  | $\left(\varphi_{2}\right)_{0}=271$ |  |
| $D G=0.30$ | $R=0.100$ | $m_{4}=5.47$ |  | $\left(\varphi_{3}\right)_{0}=37$ |  |
| $D I=0.30$ |  | $m_{5}=2.73$ |  | $\left(\varphi_{4}\right)_{0}=133$ |  |

Even though this configuration of the torque converter is not labelled as optimal, the constructive data was not selected randomly. In fact, the simulations revealed some important considerations regarding its dynamic stability. These observations refer to the case where the angular velocity of the crank is constant and no resistive torque is imposed at the output shaft. First, to ensure the stability of the CTC, the simulations exhibited that the length of the crank 1 must be small compared to the length of the other elements. In this way, the motion of the joint $C$ approaches a simple harmonic motion in a horizontal direction. It seems that the stability is increased when the joint $C$ also moves on a horizontal line passing through the fixed points $A$ and $H$. In this way, the joint $D$ of the pendulum 3 moves mainly in a horizontal direction so that the elements 6 and 8 of the mechanical diode rotate continuously in an opposite direction. If, for any configuration, the joint $D$ has a too large vertical amplitude, the branches of the mechanical diode will rotate in the same direction which may result in zero speed at the output shaft if they rotate against the allowed direction at the output. Also, if the angular speed of the motor is seen to be too large, the vertical motion of the joint $D$ results in an unstable behaviour of the CTC. The vertical motion of the joint $D$ can also be generated due to the location of the fixed point $F$. In fact, it is preferable to position the joint $F$ on the abscissa axis so that the mean position of element 4 while it oscillates is near the vertical axis. These considerations may seem intuitive, but must be put in the foreground when sizing. When the CTC drives a machine to its output shaft, the resistance is simulated by a torque distributed to the elements 6 and 8 of the mechanical diode. In fact, if the direction of rotation allowed by the unidirectional bearings is the trigonometric direction, the elements 6 and 8 will feel this resistive torque only when they rotate in the same direction. Otherwise, these elements slide without friction around the output shaft. Mathematically, the implementation of the resistive torque in the vector of generalized forces (17) is written as follows:

$$
T_{6}=\left\{\begin{array}{rr}
-T_{r}, & \text { si } \dot{\varphi}_{6}>0  \tag{19}\\
0, & \text { si } \varphi_{6}<0
\end{array} \quad T_{8}=\left\{\begin{array}{rr}
-T_{r}, & \text { si } \dot{\varphi}_{8}>0 \\
0, & \text { si } \varphi_{8}<0
\end{array}\right.\right.
$$

For a constant torque at the output shaft, the magnitude of the angular speed of the crank can result in undesirable behaviour. When this angular velocity is not chosen adequately, the motion of the pendulum 3 can bring the joint $E$ of the element 4 to stabilize in the first quadrant, which occurs because of the fact that the amplitude of the vertical motion of the joint $D$ is seen to be larger and larger. A study of the equilibrium points of the Constantinesco torque converter could be carried out in a later paper in order to obtain the conditions for which this undesirable, but still stable, position arises. This situation can also occur due to a sudden change of the resistive torque at the output shaft. In addition, as with any vibratory system, there are angular crank speeds for which the dynamic response is characterized by the phenomenon of beats. This is possible when the angular speed of the crank has a magnitude near the two natural frequencies of the CTC.

For the dynamic response of the torque converter be qualified as stable, several criteria are added to the observations mentioned previously. Among them, the variation of the kinematic parameters of all the elements must be periodic with a frequency in relation with the angular speed of the crank, in steady state, the average engine torque and the average angular speed at the output shaft are similar for each period and the variation of the motor torque as a function of the angular velocity at the output shaft follows a limit cycle.

As mentioned in the introduction, the Constantinesco torque converter has the capacity to adapt to a variation of the resistive torque at the output shaft. To observe this behaviour, the resistive torque profile of the left part of Fig. 5 is used.


Fig. 5 - Left : Distribution of the resistive torque at the output shaft ; Right : Relation between the output shaft

## velocity and the resistive torque

When the resistive torque progressively gains in magnitude, the CTC responds with a decrease in the angular speed at the output, an increase in the motor torque, but also an increase in the angular speed of the pendulum 3; he fights against the resistance. As the resistive torque becomes constant, the magnitude of the angular velocity at the output shaft also tend to be constant. For the time interval when the resistive torque decreases, the motor torque and the angular speed of the pendulum 3 decreases and the angular velocity at the output shaft increases. This simulation demonstrates the expected behaviour: the CCC can adapt in the torque-speed space. Thus, for a constant angular speed of the crank, the magnitude of the engine torque moves according to the relation shown in the right part of Fig. 5.

In the case where the output resistance is such that the Constantinesco torque converter cannot provide any angular velocity to the mechanical diode, the pendulum mainly accumulates the mechanical energy supplied by the motor until the moment the output can be put back in motion. This behaviour needs to be investigated in further simulations.

Simulations have also shown that the transmission ratio between the angular velocity at the output shaft and the crank is linear (left-hand side of Fig. 6) and that the variation of the maximum motor torque in function of the magnitude of the crank angular speed follow a second-degree polynomial (right part of Fig. 6).


Fig. 6 - Left : Maximum speed at the output ; Right : Maximum motor torque

The transmission ratio seems to be constant, and this, even if the magnitude of the resistive torque at the output shaft varies. To improve this transmission ratio, which is only approximately 0.14 , a more in-depth study of the CTC synthesis (length and mass of elements) should be put forward. As for the maximum motor torque, in practice, the latter cannot vary without limit. In fact, the curve of Fig. 6 b can be used as a tool for the selection of an appropriate motor when the angular speed of the crank is fixed.

The modelling and simulation of the Constantinesco torque converter presented in this paper represent a first step in the study of the dynamic behaviour of this continuously variable transmission. Subsequent studies on stability conditions, resonance frequency calculation and performance optimization should be considered in order to introduce the CTC in practice.

## CONCLUSIONS

This article presents a preliminary study on the dynamic behaviour of the Constantinesco torque converter. This mechanical transmission with variable ratios could be installed on agricultural tractors in order to lower the fuel consumption and reduce the number of mechanical components used to transmit power to the wheels. The general aspects of the dynamic behaviour of this converter have been presented. Future developments on the CTC's potential in the agricultural field would aim at optimizing performance resulting in a synthesis that would maximize, for example, mechanical efficiency while investigating stability conditions. Then, a more in-depth comparative study could be carried out between the use of the Constantinesco torque converter and the mechanical transmissions presently used in practice.

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