LQR AND FUZZY CONTROL FOR REACTION WHEEL INVERTED PENDULUM MODEL

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Abstract: Reaction Wheel Inverted Pendulum (RWIP) is a nonlinear system as Single Input Multiple Output (SIMO). In order to control pendulum of RWIP at the upright position, reaction wheel must be controlled appropriately. In this paper, authors propose Linear Quadratic Regulator (LQR) algorithm and Fuzzy Logic Controller (FLC) for controlling RWIP. Results from the simulation are tested on Matlab/Simulink Tool. Then, authors implement the plant on real model. LQR and FLC can control pendulum of RWIP which holds on upright position when system is affected by external force.

Keywords: Reaction Wheel Inverted Pendulum (RWIP), nonlinear, upright position, Linear Quadratic Regulator (LQR), Fuzzy Logic Controller (FLC).

1. Introduction

The systems of balancing control are interested more and more in the science research. Future vehicles such as self-balancing bicycle, self-balancing motorbike, etc. are researched and made by manufacturers such as Honda, Yamaha, BWM, etc. In order to control the balancing above vehicles, engineers and designers have used control algorithms. Algorithms are mentioned which can be executed: PID, Fuzzy, etc. In this paper, RWIP system [1] is a nonlinear plant can be used for testing algorithms. The structure of RWIP included pendulum and reaction wheel. If RWIP is not controlled as pendulum, it will fall down. In order to balance this kind of pendulum, the reaction wheel has to move in rotary direction with suitable speed. Although the pendulum is affected by external force, it has been balanced.

Control algorithms such as PID [2], sliding mode [3], fuzzy [4], etc. are very popular in laboratory. In this paper, authors propose two controllers for balancing RWIP. Firstly, authors use LQR [5] algorithm to calculate the value of control signal for reaction wheel. Secondly, authors use FLC to control this system (RIWP) through training based on LQR.

Researching on how to balance RWIP helps engineers to design self-balancing bike or motorbike. Indeed, Fig. 1 [6] may prove this description.

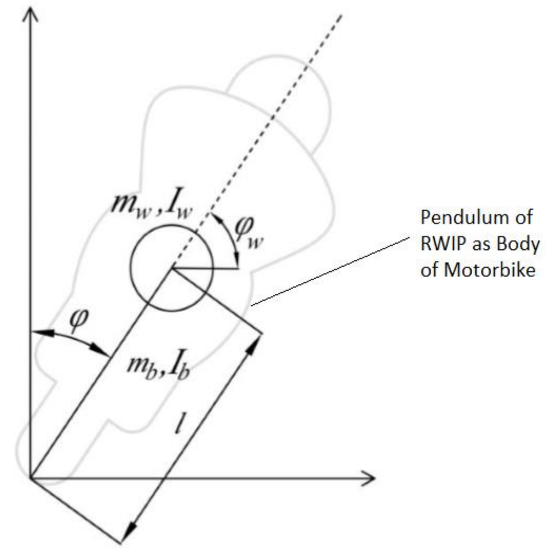

Fig. 1. Relationship between RWIP and self-balancing motorbike.

According to Fig. 1, it is easy to see that the pendulum as body of motorbike and the circle is reaction wheel which is controlled by a motor DC. Authors are going to mention the calculating voltage and rotary direction for motor DC in main contents.

In section 2, authors describe dynamic equations of RWIP system. In section 3, LQR is designed. In section 4, authors design the Fuzzy Logic algorithm in order to compare with LQR. In section 5, the simulation results and the experimental results are shown. In section 6, the conclusion ends the paper.

2. Dynamic equations of System

Similar to others nonlinear system such as Inverted Pendulum [7], Acrobot system [8], RWIP is generated through Euler-Larange method. Descartes coordinate system was chosen in Fig. 2 [9] as follow:

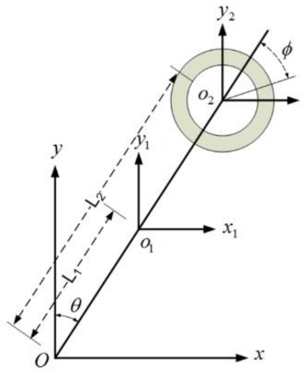

Fig. 2. Mathematical model of RWIP.

Parameters of RWIP in Table 1 are recorded by actual measurement on model. Variables are θ, ϕ and T_r. And, θ and ϕ are output signals; T_r is input signal. Authors can find out the relationship among input and output signals refer from algorithm.

Tab. 1

Parameters and variables	Description
L_1	Length of Pendulum from O to O_1
L_2	Length of Pendulum
m_1	Mass of Pendulum
m_2	Mass of Wheel
θ	Angle of Pendulum
ϕ	Angle of Wheel
I_1	Inertia moment of Pendulum
I_2	Inertia moment of Wheel
g	Gravitational acceleration
T_r	Torque applied by DC Motor

Lagrange Method is obtain as [1]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=\tau_i \tag{1}$$

Where L is Lagrange equation that is determined by (2):

$$L\left(q,\dot{q}\right)=K(q,\dot{q})-V(q,\dot{q}) \tag{2}$$

$K(q,\dot{q})$ is Kinetic energy and $V(q)$ is Potential energy; τ_i: total forces affecting to system; q: components of system.

From Fig. 1, Kinetic energy and Potential energy of system are defined by (3) and (4):

$$K=\frac{1}{2}(m_1L_1{}^2+m_2L_2{}^2+I_1+I_2)\dot{\theta}^2+I_2\dot{\theta}\dot{\phi}+\frac{1}{2}I_2\dot{\phi}^2 \tag{3}$$

$$V=(m_1L_1+m_2L_2)g\cos\theta \tag{4}$$

From (3) and (4), Lagrange equation is defined based on (2). Next, the calculating follows as (1), the mathematical equations of RWIP is described as:

$$\begin{aligned}&(m_1L_1{}^2+m_2L_2{}^2+I_1+I_2)\ddot{\theta}+I_2\ddot{\phi}\\&-(m_1L_1+m_2L_2)g\theta=0\end{aligned} \tag{5}$$

$$I_2(\ddot{\theta}+\ddot{\phi})=T_r \tag{6}$$

State Space equation is determined by (5) and (6) with control signals are moment:

$$\begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix}=\begin{bmatrix}0\,1\,0\,0\\ b/a\;0\;0\;0\\ 0\,0\,0\,1\\ -b/a\;0\;0\;0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ \phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ -1/a\\ 0\\ (a+I_2)/(aI_2)\end{bmatrix}T_r \tag{7}$$

where:

$$a=m_1L_1{}^2+m_2L_2{}^2+I_1,b=(m_1L_1+m_2L_2)g \tag{8}$$

In order to control DC motor easily, authors converted moment to voltage. The relationship between voltage and moment is described through [9]:

$$V=L_m\frac{di}{dt}+R_mi+K_e\omega_m \tag{9}$$

$$T_m=K_ti \tag{10}$$

$$T_r=N_gT_m \tag{11}$$

Tab. 2

Parameters and variables	Described
V	Applied voltage for DC motor
K_e	Moment constant of DC motor
ω_m	Angular velocity of DC motor
L_m	Inductor of DC motor
R_m	Resistor of DC motor
i	Current flow through DC motor
T_m	Generated moment of DC motor
K_t	Constant of DC motor
N_g	Transmission ratio of DC Motor

where $L_m << R_m$, (9) can be written follow as:

$$V=R_mi+K_e\omega_m \tag{12}$$

The relationship between motor speed and wheel speed is shown as in (13):

$$\text{Let}\begin{cases}\omega_r = \dot{\phi} \\ \omega_m = N_g \omega_r\end{cases} \quad (13)$$

where ω_r is angular velocity of wheel.

From (9) to (13), relationship between applied voltage for DC motor and the moment of DC motor is

$$T_r = N_g K_t \left(\frac{V - K_e N_g \dot{\phi}}{R_m}\right) \quad (14)$$

From (7) and (14), mathematical equations of RWIP are

$$\begin{cases}\begin{bmatrix}\dot{\theta} \\ \ddot{\theta} \\ \dot{\phi} \\ \ddot{\phi}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0 \\ a_{21} & 0 & 0 & a_{24} \\ 0 & 0 & 0 & 1 \\ a_{41} & 0 & 0 & a_{44}\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ \phi \\ \dot{\phi}\end{bmatrix} + \begin{bmatrix}0 \\ b_2 \\ 0 \\ b_4\end{bmatrix} V \\ y = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta & \dot{\theta} & \phi & \dot{\phi}\end{bmatrix}^T\end{cases} \quad (15)$$

where:

$$a_{21} = \frac{b}{a}, a_{24} = \frac{K_t K_e N_g{}^2}{a R_m},$$
$$a_{41} = -\frac{b}{a}, a_{44} = -\left(\frac{a + I_2}{a I_2}\right)\left(\frac{K_t K_e N^2{}_g}{R_m}\right), \quad (16)$$
$$b_2 = -\frac{K_t N_g}{a R_m}, b_4 = \left(\frac{a + I_2}{a I_2}\right)\frac{K_t N_g}{R_m}.$$

The State Space variables are:

$$\begin{cases}x_1 = \theta \\ x_2 = \dot{\theta} \\ x_3 = \phi \\ x_4 = \dot{\phi}\end{cases} \Rightarrow \begin{cases}\dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(x) + b_1(x)u \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = f_2(x) + b_2(x)u\end{cases} \quad (17)$$

where:

$$\begin{cases}f_1(x) = a_{21}x_1 + a_{24}x_4 \\ b_1(x) = b_2 \\ f_2(x) = a_{41}x_1 + a_{44}x_4 \\ b_1(x) = b_2\end{cases}$$

3. Linear Quadratic Regulator Algorithm

The system is described continuous-time as follow [2]:

$$\dot{x} = Ax + Bu$$

where:

$$A = \begin{bmatrix}\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \cdots \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \cdots \frac{\partial f_2}{\partial x_n} \\ \cdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \frac{\partial f_n}{\partial x_3} & \frac{\partial f_n}{\partial x_4} \cdots \frac{\partial f_n}{\partial x_n}\end{bmatrix}, B = \begin{bmatrix}\frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \cdots \\ \frac{\partial f_n}{\partial u}\end{bmatrix} \quad (18)$$

Adaptive Function is determined as in (21):

$$I = \int_0^\infty (x^T Q x + u^T R u + 2x^T N u) dt \quad (19)$$

where: Q is matrix with n x n size; R is weighting matrix:

$$Q = \begin{bmatrix}q_1 & 0 & 0 & 0 \ldots 0 \\ 0 & q_2 & 0 & 0 \ldots 0 \\ \ldots \\ 0 & 0 & 0 & 0 \ldots q_n\end{bmatrix}, R = r \quad (20)$$

The control signal designed by LQR is

$$u = -Kx \quad (21)$$

where:

$$K = R^{-1}(B^T P + N^T) \quad (22)$$

P is calculated by Riccati function as:

$$A^T P + PA - (PB + N)R^{-1}(B^T P + N^T) + Q = 0 \quad (23)$$

In all cases, we can set N=0 to simply calculate Riccati function. In order to find K, authors use below command on Command Window of Matlab Software:

$$K = dlqr(A, B, Q, R) \quad (24)$$

where matrices A, B and Q, R are determined at (20), (22).

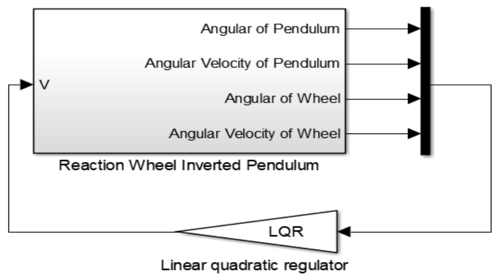

Fig. 3. Scheme of LQR controller for RWIP.

Parameters of model are in Table 3:

Tab. 3

Parameters	Values
m_1	0.96 Kg
m_2	0.35 Kg
L_1	0.11 m
L_2	0.18 m
K_t	0.0649(Nm/A)
K_e	0.0649(Vs/rad)
N_g	1
R_m	6.83 Ω
I_1	0.0212 Kg. m^2
I_2	0.0027 Kg. m^2

With parameters in Table 3, matrix A and B of state space model are calculated as:

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 37.4259 & 0 & 0 & 0.0140 \\ 0 & 0 & 0 & 1 \\ -37.4259 & 0 & 0 & -0.2396 \end{bmatrix}, B=\begin{bmatrix} 0 \\ -0.2151 \\ 0 \\ 3.6895 \end{bmatrix} \quad (25)$$

Weight matrix Q and R are chosen for LQR controller as (28):

$$Q=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, R=1 \quad (26)$$

Matrix K is determined for feedback control:

$$K=[-601.0636 \;\; -99.5773 \;\; -0.9211 \;\; -1.5320] \quad (27)$$

Hence, LQR controller is designed with above vector K.

4. Fuzzy Logic Control

Fuzzy Logic Control (FLC) is a classical algorithm which controls many industrial systems. FLC is applied in machines such as Washing Machine, Air Conditioner, etc. Quality of FLC depends on the designer's experience. In this paper, authors also design a Fuzzy controller to control RWIP. Fuzzy controller is designed based on results from LQR controller in order to control system.

In this paper, FLC has four inputs and one output. In order to train FLC is like LQR, authors make output of FLC follow as LQR output. It means that output of FLC will be designed as in (22). Authors use Adaptive Neuron Fuzzy Inference System (ANFIS) toolbox to get input and output data. After that, the data is trained to define membership functions. Finally, FLC which contains inputs and output is given.

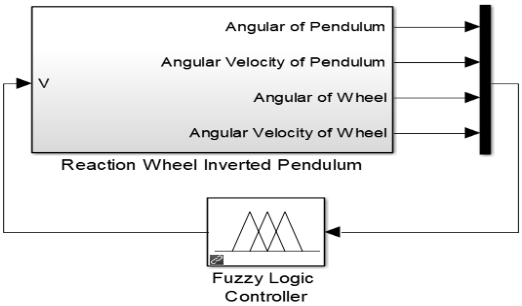

Fig. 4. Scheme of FLC for RWIP.

Fuzzy algorithm is used by authors in this paper which defined by J.-S. Roger Jang in 1992. Follow as Roger Jang proposes who creates a fuzzy decision tree to classify the data into one of 2n (or pn) linear regression models to minimize the sum of squared errors (SSE) [3]:

$$SSE=\sum_j e_j^{\,2} \quad (28)$$

where:

e_j : error between reference output and real output

p: number of fuzzy partitions of each variable

n: number of input variables

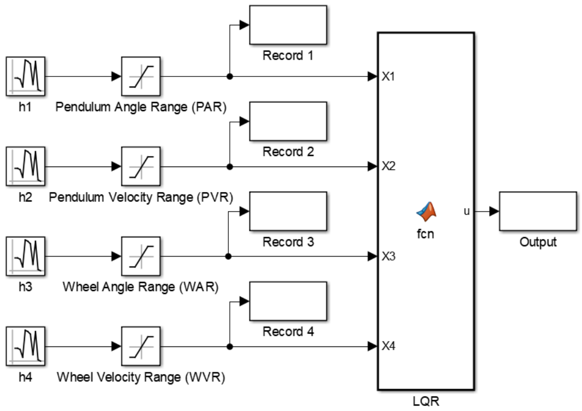

Fig. 5. Scheme for identifing LQR.

$h1, h2, h3, h4$ are free input variables for taking samples. Based on results from experiment, authors limit input variables as follow:

$$PAR=[-10^\circ;10^\circ], PVR=[-7\frac{rad}{s};7\frac{rad}{s}]$$

$$WAR=[-180^\circ;180^\circ], WVR=[-7\frac{rad}{s};7\frac{rad}{s}]$$

5. Simulation results and experimental results

RWIP is controlled stably by LQR and FLC. Results are shown in Fig. 5-9. Initial values of system are $\theta=4(\text{deg}), \phi=0(\text{rad}), \dot{\theta}=0(\text{rad/s}), \dot{\phi}=0(\text{rad/s})$.

Fig. 6. RWIP model when being unstable.

Fig. 7. RWIP model when being stable..

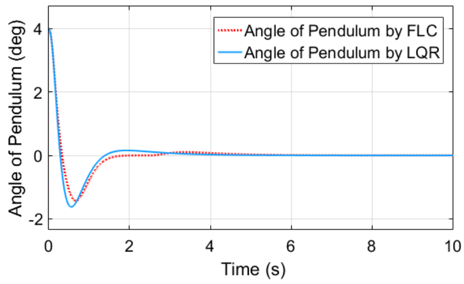

Fig. 8. Angle of Pendulum response (Simulation).

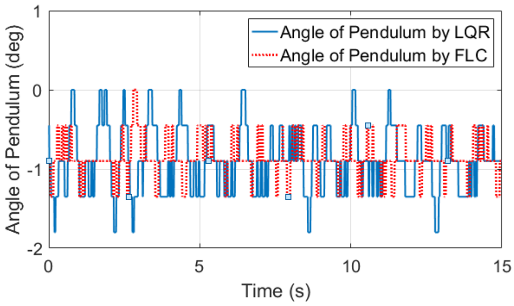

Fig. 9. Angle of Pendulum response (Experiment).

In Fig. 7 and Fig. 8, Pendulum angle is compared between LQR and FLC. Viewers may see system under FLC is more stable than under LQR. Response of FLC is little changed in control process. However, FLC gives limit range of initial value is not over 4 degrees during LQR gives 5 degrees.

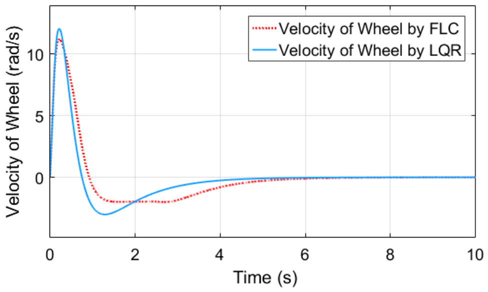

Fig. 10. Angular Velocity of Wheel response (Simulation).

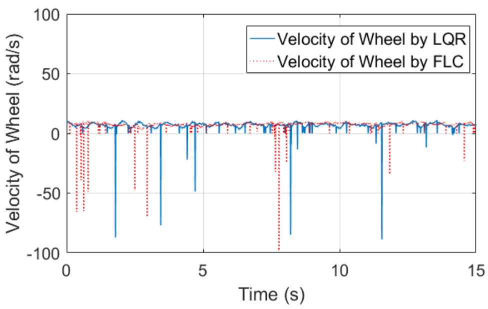

Fig. 11. Angular Velocity of Wheel response (Experiment).

In Fig. 9 and Fig. 10, the velocity of wheel is also mentioned by both simulation and experiment. Strength of FLC decreases overload of the velocity of wheel. Strength of LQR decreases setting time. Hence results from experiment show that LQR is as quality as FLC.

6. Conclusions

LQR controller is proposed by authors that controlled stably RWIP through both simulation and experiment. Beside, FLC shows higher ability for controlling RWIP system. An important problem should be cared when control balance for RWIP is initial values. Initial value is position of pendulum which will be set before system operates. If pendulum is not in limited range, system is unstabe. Although initial values of FLC is smaller than initial values of LQR, FLC can help system decrease overload. In order to get results, FLC must be trained suitably.

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