

APPLICATIONS OF THE GAME THEORY IN THE MANAGERIAL SYSTEM

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Abstract: The article analyses applications of the game theory in the context of the new applied microeconomics, highlighting the implications of this approach on market decisions in monopolistic and duopolistic situations. The tendency towards equilibrium of any economic system is the key to using game theory to identify price levels and output volumes in different forms of competition, the differentiation being brought about by the number and economic power of competitors.

Keywords: Market, competition, economic game, decision, equilibrium.

1. Introduction

The game theory is based on the equilibrium in any economic system. Namely, on the fact that the sum of all losses is equal to the sum of all winnings. Each participant depends on the decision that all the other players make. Thus, the possible decisions of others must be taken into account.

The game theory is a relatively new branch of microeconomics developed over the last 60 years. The first general theorem is forwarded by E. Zermelo in 1913. But during the period between the two world wars the contributions of the mathematicians Emile Borel and von Neumann laid the foundations for the theory. The technique used in the analysis of these situations was developed by the mathematician John von Neumann. At the beginning of the 1940s he worked together with economist Oskar Morgenstern in the economic applications of this theory. The book they published in 1944, "Game Theory and Economic Behavior", opened an unexpected wide field of study in which thousands of specialists throughout the world work today. The game theory has achieved a high degree of mathematical sophistication and has shown great versatility in solving problems [1].

In general, small and medium organizations are forced, for any type of building strategy or any change of its status, to obtain loans, subventions or non-refundable funds, to realize managerial analyses, from which the simplest and most useful is the SWOT analysis [2].

2. Game Theory - Concept

The game can be defined as a formal representation of a situation where a number of n individuals interact within a strategic interdependence. The essential elements of a game are the players, the actions they take, the winnings, that is the best result on the level of each player and the result or the information

obtained from each possible combination of strategies [3].

A player's strategy is a feasible action that the player can choose in the game. The game's strategy is given by the crowd of players' strategies. So the set of game strategies is $S = S_1 \times S_2 \times \dots \times S_n$, where n is the number of players.

The function of game winning, namely $u = (u_1, u_2, \dots, u_n)$ is made of the winning functions of each player. Thus, if the winning function of each player is u_i and the winning functions of the other players are u_{-i} , the game winning function will be: $u : S \rightarrow R, u = (u_i, u_{-i})$.

An $s^* = (s_1^*, \dots, s_n^*)$ equilibrium is a combination of strategies made of the best strategy for each of the n players.

A game in a normal or strategic form is defined by three elements: the set of players $i \in \Pi$, with $\Pi = \{1, 2, \dots, I\}$ a finite set, the space of pure strategies S_i and for each player i and the win or pay functions u_i . As a definition attributed to pure strategy $s_i \in S_i$ we can say that this represents for the player i the achievable action that can be chosen by the latter from the pure strategy space S_i which will bring the player the winning $u_i(s)$. The second element of the game in normal form is the vector of the achievable actions chosen by the players at one point. Last but not least, the winning functions $u_i(s)$ are defined as von Neumann - Morgenstern utility functions for each profile of achievable strategies $s = (s_1, \dots, s_i)$, that is, depending on the strategies chosen by all the players [4].

Identifying as well as becoming aware of the company's growth potential represents, especially under the crisis and recession circumstances, an obligation for the strategic approach of the firm's management.

Strategic objectives and corresponding goals are developed based on a very thorough assessment of the organization and the external environment [5].

3. Game Theory Applications in Low-Competition Markets

3.1. The Game of Bilateral Monopoly

A firm is in a monopoly situation when the cross-price elasticity of demand in the product it sells, in relation to the price at which the same product sells all the other firms, is zero or close to zero. [6]

The bilateral monopoly is the market situation where there is only one supplier and one buyer. If we assume that the offeror and the buyer are represented by two companies, the two partners or players in terms of game theory, must decide on the price p_1 and the amount of exchange y_1 . In this case $C_1(y_1)$ represents the production cost of y_1 for the offeror, $R_2(y_1) - p_1 y_1$ representing the profit the demand bearer will have when the quantity is used y_1 , and the winnings of the two participants have the form: $W_1 = p_1 y_1 - C_1(y_1)$ and $W_2 = p_2 y_2 - C_2(y_2)$ respectively.

To determine the winning functions of the type of those used in game theory, it is necessary to specify the a_1 and a_2 activities performed by the two companies and the afferent domains of activity A_1 and A_2 . Assuming that the first firm or player A sets the price p_1 , and player B sets the amount it receives y_1 , then A_1 and A_2 are defined by $p_1 > 0$ and $y_1 > 0$. If the price p_1 is set by firm A, then the firm shall choose quantity y_1 , so that $R_2'(y_1) = p_1$. Considering that the quantity y_1 is given then the first player is interested in setting the highest price possible. The only possible non-cooperative equilibriums correspond to $y_1 = 0$ and $p_1 > R_2'(0)$, namely of a null production of the respective merchandise.

In this case, company A attempts to obtain a maximum of its profit taking into account the amount fixed by its partner. Thus, it produces the quantity y_1^* : $C_1'(y_1) - y_1 R_2''(y_1) = R_2'(y_1)$ and sells at the price $p_1^* = R_2'(y_1^*)$. Thus, the two companies must agree, either by concluding an immediate agreement or after a few exploratory actions. For this reason, it is of little importance if the first one sets p_1 and the second the quantity y_1 , because it is intended to achieve a satisfactory (p_1, y_1) combination.

This satisfactory combination should meet the following conditions:

1. it must lead to a value W_1 at least equal to $-C_1(0)$ otherwise company A would lose interest and abandon any exchange with company B;
2. it must lead to a value W_2 at least equal to $R_2(0)$;
3. it must maximize W_1 under the restriction W_2 , preserving the value W_2^0 , because company A could suggest to company B a combination that would be satisfactory for both A and B;

4. it must maximize W_2 under the restriction that W_1 preserves the value W_1^0 .

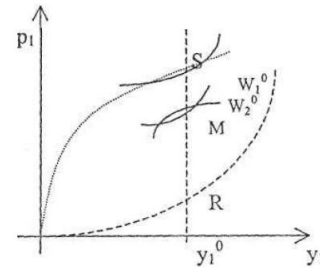


Fig. 1. The game of bilateral monopoly.

It can be seen that in the node (represented by the RS interval), the interests of the two partners are strictly opposite. Each of them has the possibility to exercise a threat of agreement non-compliance in the hope that the other combatant will accept its requirements. But none of the two possesses a threat that would surely guarantee it a higher win than the one it would achieve in the absence of the exchange. In this respect, the non-cooperative equilibrium is not a solution for the bilateral monopoly. Therefore, partners have an interest in agreeing in order to achieve one of the combinations belonging to the node.

3.2. The Cournot Duopoly

The Cournot-type equilibrium is obtained by determining the best reply of each player, given the amount produced by the other player.

To exemplify a Cournot equilibrium in a graph we can assume that there is a market in which two firms operate. For the demand curve $Y(p) = 1000 - 1000p$, we have an inverse demand curve of $p(Y) = 1 - 0.001Y$ and a marginal cost equal to $C_m = 0.28$. Each of the two companies produces the way to respond to global demand. The first of the two companies is faced with a residual claim: $y_1 = Y(p) - y_2$. The calculations are obtained $y_1 = 360 - y_2/2$ and $y_2 = 360 - y_1/2$.

From fig. 2 we may remark that the intersection between the optimal response curves of the two companies represents the Cournot-Nash equilibrium. In this case none of the two companies wants to change its choice [6].

If the two companies act as one, they can reach an agreement to maximize the global profit. This maximum global profit is the one within a monopoly.

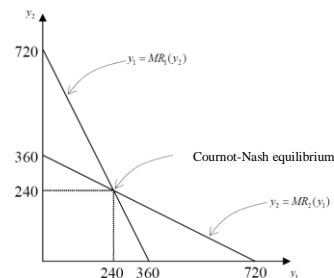


Fig. 2. The Cournot equilibrium.

With regard to the above example, we consider that the two companies have the opportunity to choose two production levels, either 180 or 220, i.e. the production level for which the Nash equilibrium was achieved. If each company wants to choose its own strategy or action without knowing how the other works, then we are in the category of games in imperfect information. Suppose the production level of an enterprise $y_n = 220$ when the Nash equilibrium is reached, and if the two form a monopoly, a production level $y_m = 180$.

Table 1 Data on production and profit

Production level	Total production	Profit per unit produced
220, 220	440	0,25
220, 180	420	0,30
180, 180	360	0,35

In this case, the dominant strategy leads the two firms to obtain a lower return than the one won if they had cooperated. The Nash equilibrium obtained (Y_n, Y_m), from the matrix in fig. 3, does not maximize its profits.

		Company 2	
		$Y_n = 220$	$Y_m = 180$
Company 1	$Y_n = 220$	(55;55)	(66;54)
	$Y_m = 180$	(54;66)	(63;63)

Fig. 3. The results matrix.

The two companies coordinate their actions well, being interested in agreeing to set a lower production level. Thus, through the Cournot equilibrium, a lower level of production can be determined than the one obtained in the perfect competition situation and much higher than in the case of monopoly. For this reason, at the limit, the Cournot equilibrium tends to the perfect competition situation when the number of firms tends to infinity. The practice of the game theory leads to the investigation of these situations as a Nash equilibrium [4].

The equilibrium for which the players' strategies are those about quantity is also a Cournot equilibrium, which is why it is called the Cournot-Nash equilibrium. By precisely defining strategies and decision sequences, and using the game theory toolkit, it is possible to interpret the Cournot equilibrium as an equilibrium of a non-cooperative game. The Cournot model shows that the competition system is not necessarily a procedure for allocating resources effectively, because in the situation of imperfect competition the level of production is inferior to that earned in the case of perfect competition.

4. The Game of Price Adjustment

Among the possible applications of game theory in the economy are the analysis of dynamic price adjustment. For example, a company operates under the slogan "no one will sell cheaper than us", and its biggest rival is company 2 that will launch in a promotional $P_{\text{Price of firm B}}$ and will make public the strategy: "we are practicing 20% price cuts". This is plotted in a graph in Figure 4 [8].

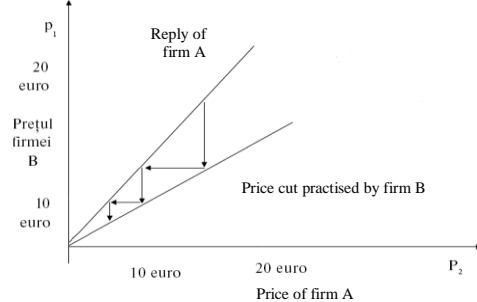


Fig. 4. The game of dynamic price adjustment.

Figure 4 shows the results obtained by the two companies engaged in the game of duopoly price. The vertical arrows correspond to the price reductions applied by company 2, while the horizontal ones of the company's reaction 1 for each price reduction. Following the way in which the two firms react, it is noticed that rivalry between them can cause mutual destruction, because the only price comparable to their strategies is the null price. It may be remarked that every time company 2 practices the strategy of price cut, the rival reacts in the same way. The elements presented above can be translated into a classic example of duopoly price war solved by the game theory. In this situation, there are only two firms on the market who decide on their own whether or not to engage in an economic war consisting of successive price cuts.

In order to simplify, we shall assume that the structure of costs and of demand is identical for each of the two companies. Also, each of them can choose either to practice the normal price or to lower the price below marginal costs in an attempt to make the rival go bankrupt. In the duopoly game, the size of the profits of one firm depends both on the strategy adopted by the other firm and on its own strategy. In order to better observe the interaction between the two players, the game results matrix will be used.

		Price of company 1	
		Normal price	Price war
Price of company 2	Normal price	(25,25)	(250,25)
	Price war	(25,250)	(150,150)

Fig. 5. The results matrix.

In the matrix in fig. 5 the results afferent to the different strategies of the two companies are presented.

The companies have to choose between the strategy of practicing a normal price and that of starting a price war. For example, if company 2 plays the strategy of a price war and company 1 plays the strategy at a normal price, the result of the game is as follows: company 2 obtains a profit of 250 and the competitor a profit equal to 25.

5. Conclusions

In the informational society, created after 1981, everything in the universe is spreading, energy is dissipated and from too much information organised disorder is created. The same happens in business if a lot of firms develop in diverse industries and you want to do it all, products and services for all market segments. [9]

The game theory can be used as a modern working tool in economy, in analysing the inequality of exchanges between agents, exchanges that are inevitably subjected to transaction costs. Thus, the game theory provides economic agents with a set of possible strategies or game scenarios to simulate different market structures and especially to formulate and adopt decisions leading to the best possible outcome.

Although at first glance the cooperative-type results are preferred, in which each economic agent tries to determine the game's solution by invoking the players' beliefs, the cooperative approach does not exhibit too much interest, as the players are not sanctioned in the event of agreement non-compliance. As shown in this paper, in fact microeconomic analysis, in terms of game theory, attaches importance to the Nash-type non-cooperative equilibriums because things are totally different from a non-cooperative point of view, the conclusion of agreements is ensured by observing them because the players are directly interested in this. For this reason, in the microeconomic analysis of price issues, it is focused on the hypothesis that each economic

agent aims to maximize its own gain in a non-cooperative approach.

In general, it can be appreciated that a game is any situation where more decision makers are called upon to make decisions that lead to results, often different, each player in the capacity of decider obtains a result, but the latter depends on the decisions made by the others.

6. References

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Personal Notes

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